MISES, KANTOROVICH AND ECONOMIC COMPUTATION

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1. What is economic calculation?

In contemporarary society the answer seems simple enough: economic calculation involves adding up costs in terms of money. By comparing money costs with money benefits one may arrive at a rational - wealth maximising - course of action.

In a famous paper[von Mises(1935)] the Austrian economist Mises argued that it was only in a market economy in which money and money prices existed, that this sort of economic rationality was possible.

His claims were striking, and, if they could be sustained, apparently devastating to the cause of socialism. The dominant Marxian conception of socialism involved the abolition of private property in the means of production and the abolition of money, but Mises argued that "every step that takes us away from private ownership of the means of production and the use of money also takes us away from rational economics" ([von Mises(1935)]: 104). The planned economy of Marx and Engels would inevitably find itself "groping in the dark", producing "the absurd output of a senseless apparatus" (106). Marxists had counter-posed rational planning to the alleged ‘anarchy’ of the market, but according to Mises such claims were wholly baseless; rather, the abolition of market relations would destroy the only adequate basis for economic calculation, namely market prices. However well-meaning the socialist planners might be, they would simply lack any basis for taking sensible economic decisions: socialism was nothing other than the "abolition of rational economy".

As regards the nature of economic rationality, it is clear that Mises has in mind the problem of producing the maximum possible useful effect (satisfaction of wants) on the basis of a given set of economic resources. Alternatively, the problem may be stated in terms of its dual: how to choose the most efficient method of production in order to minimize the cost of producing a given useful effect. Mises repeatedly returns to the latter formulation in his critique of socialism, with the examples of building a railway or building a house:[1] how can the socialist planners calculate the least-cost method of achieving these objects?

As regards the means for rational decision-making, Mises identifies three possible candidates: planning in kind (in natura), planning with the aid of an ‘objectively recognizable unit of value’ independent of market prices and money, such as labour time, and economic calculation based on market prices. I will go on to examine Mises’, very influential, arguments in section 2, but first I will examine whether an alternative interpretation can be placed on the concept of economic calculation.

[1]The railway example is in [von Mises(1935)]. The house-building example is in Human Action [von Mises(1949)].

It is clear that monetary calculation lends itself well to problems of the minimising or maximising sort. We can use money to find out which of several alternatives is cheaper, or which sale will yield us the most profit. But if we look in more detail at what is involved here, we shall see that a lot of calculation must take place prior to money even being brought into consideration. Let us look not at building a mere house, but at something grander, the first Pyramid at Saqqara, planned by Imhotep[Bierbrier(1989)]. In order to build this Imhotep had to carry out a whole mass of calculations. He needed, for example, to know how to calculate the volume of pyramid before it was built([Lumpkin(1997)], p.40), which involves a fair degree of sophisticated geometry[2]. From a knowledge of the volume of a pyramid, and a knowledge of the size of the stones he planned to use, he could calculate how many stones would be required. Knowing the rate at which stonemasons could put the stones in place he could estimate how long it would take workforces of different sizes to place all the stones for the pyramid. From the number of stones too, and knowledge of how many people are needed to transport each stone, Imhotep could work out the number of people who would have to work shifting the stone from the quarry to the pyramid.

This workforce would have to be fed, so bakers, brewers and butchers were needed to feed them ([David(1996)],ch.6). He, or his scribes, would have to calculate how many of these tradesmen were required. Quantities of grain and cattle would have to to be estimated. In the broadest sense, this was all economic calculation, but it would have taken place without money, which had yet to be invented. It might be objected that this is not what Mises meant by economic calculation, since Imhotep's calculation 'in kind' was not economic calculation but engineering calculation, a mere listing of prerequisites, what was missing was the valuation or costing of these inputs. Fair enough, this is not what von Mises meant by economic calculation, the question is, whether he was right to limit this concept to monetary calculation. Imhotep's calculations do reveal that Mises concept may have been too narrow. Suppose that the pyramid were built now, a large part of the calculations required would be the same. It would still be necessary to work out how much stone would be used, how much of various types of labour would be used, how the stone was to be transported etc. This would be the difficult part of the calculation, totalling it up in money would be easy in comparison.

Consider the issue of choosing between the most economical alternative. Imhotep certainly had to address this question. Building a pyramid was, even by modern standards, a massive undertaking. To complete it he not only had to address questions of structural stability but he also had to devise a practical method by which stones could be raised into place. That this was no easy task is born out by the fact that we still do not know for sure how it was done. Various suggestions have been made: sloping ramps at right angles to the pyramid wall up which stones were hauled; spiral ramps wrapping round the pyramid; internal tunnel ramps; a series of manually operated cranes; etc. If we today can think

[2]The Rhind Papyrus, the earliest known collection of mathematical problems, includes examples where the student had to calculate the volume of, and thus the number of bricks required for, pyramids.

of lots of possible ways in which it might be done, so to, we can assume, must the original builders, before settling on whatever method that they actually used. The resources of manpower available to them were not unlimited, so they had to discover an approach that was both technically feasible and economically feasible. This is the sort of rational choice that Mises saw as impossible without money, but the fact that the pyramids were built, indicates that some calculation of this type did occur.

The ultimate constraint here was the labour supply available; no sensible architect would embark on a course of construction that used far more labour than another. In a pre-mercantile economy like ancient Egypt this labour constraint appears directly, in a mercantile economy, the labour constraint appears indirectly in the form of monetary cost. The classical political economists argued that money relations disguised underlying relations of labour, money costs hid labour costs; money was, for Adam Smith, ultimately the power to command the labour of others.

2. Planning in kind

The organisational task that faced a pyramid architect was vast. That it was possible without money was an indication that monetary calculation was not a sine qua non of calculation. But as the project being planned becomes more complex, then planning it in material units will become more complex. Mises is in effect arguing that optimization in complex systems necessarily involves arithmetic, in the form of the explicit maximization of a scalar objective function (profit under capitalism being the paradigmatic case), and that maximising the money return on output, or minimising money cost of inputs is the only possible such scalar objective function. Mises argued for the impossibility of of planning in kind because, he said,. the human mind is limited in the degree of complexity that it can handle.

So might the employment of means other than a human mind make possible planning in kind for complex systems?

There are two 'inhuman' systems to consider:

(1) Bureaucracies. A bureaucracy is made up of individual humans, but by collaborating on information processing tasks, they can carry out tasks that are impossible to one individual.
(2) Computer networks. Nobody familiar with the power of Google[3] to consolidate and analyse information will need persuading that computers can handle volumes and complexities of information that would stupefy a single human mind, so a computer network could clearly do economic calculations far beyond an individual human mind.

More generally as Turing pointed out [Turing(1936)] any extensive calculation by human beings depends on artificial aides-memoir, papyrus, clay tablets, slates, etc. With the existence of such aides to memory, algorithmic calculation becomes possible, and at this point the difference between what can be calculated by a human using paper and pencil methods

[3]The algorithms used by Google involve the solution to large sparse systems of linear equations. This, as we shall see later, is the same type of calculation as is required for planning in kind.

or a digital computer come down only to matters of speed[Turing(1950), Turing(2004)]. There is thus no difference in principle between planning using a bureaucracy and planning using computers, but there is in practice a big difference in the complexity of problem that can be expeditiously handled.

There is no question that the procedure of economic calculation considered by von Mises was primarily algorithmic. It involves a fixed process of

(1) For each possible technique of production
    (a) form a physical bill of materials,
    (b) use a price list to convert this into a list of money expenditures,
    (c) then add up the list to form a final cost
(2) Select the cheapest final cost out of all the costs of techniques of production

The question then arises as to whether there exists *in-natura* algorithms with an analogous function?

2.1. **Kantorovich's method.** In the 20s and early 30s when Mises first advanced his arguments, no such algorithmic techniques were known. But in 1939 [Kantorovich(1960)] the Soviet mathematician V Kantorovich came up with a method which later came to be known as *linear programming* or *linear optimisation,* for which he was later awarded both Stalin and Nobel prizes. Describing his discovery he wrote:

> I discovered that a whole range of problems of the most diverse character relating to the scientific organization of production (questions of the optimum distribution of the work of machines and mechanisms, the minimization of scrap, the best utilization of raw materials and local materials, fuel, transportation, and so on) lead to the formulation of a single group of mathematical problems (extremal problems). These problems are not directly comparable to problems considered in mathematical analysis. It is more correct to say that they are formally similar, and even turn out to be formally very simple, but the process of solving them with which one is faced [i.e., by mathematical analysis] is practically completely unusable, since it requires the solution of tens of thousands or even millions of systems of equations for completion.
>
> I have succeeded in finding a comparatively simple general method of solving this group of problems which is applicable to all the problems I have mentioned, and is sufficiently simple and effective for their solution to be made completely achievable under practical conditions. ([Kantorovich(1960)], p. 368)

What was significant about Kantorovich's work was that he showed that it was possible, starting out from a description in purely physical terms of the various production techniques available, to use a determinate mathematical procedure to determine which combination of techniques will best meet plan targets. He indirectly challenged von Mises[4], both

[4]There is no indication that he was aware of von Mises at the time.

TABLE 1. Kantorovich's first example.

Type of machine	# machines	output per machine		Total output	
		As	Bs	As	Bs
Milling machines	3	10	20	30	60
Turret lathes	3	20	30	60	90
Automatic turret lathes	1	30	80	30	80
Max total				120	230

TABLE 2. Kantorovich's examples of output assignments.

Type of machine	Simple solution		Best solution	
	As	Bs	As	Bs
Milling machines	20	20	26	6
Turret lathes	36	36	60	0
Automatic turret lathes	21	21	0	80
Total	77	77	86	86

by proving that in-natura calculation is possible, and by showing that there can be a non monetary scalar objective function : the degree to which plan targets are met.

The practical problems with which he was concerned came up whilst working in the plywood industry. He wanted to determine the most effective way of utilising a set of machines to maximise output. Suppose we are making a final product that requires two components, an A and a B. Altogether these must be supplied in equal numbers. We also have three types of machines whose productivities are shown in the Table 1.

Suppose we set each machine to produce equal numbers of As and Bs. The three milling machines can produce 30 As per hour or 60 Bs per hour. If the 3 machine produce As for 40 mins in the hour and Bs for 20 mins then they can produce 20 of each. Applying similar divisions of time we can produce 36 As and Bs on the Turret lathes and 21 As and Bs on the automatic turret lathe (Table 2).

But Kantorovich goes on to show that this assignment of machines is not the best. If we assign the automatic lathe to producing only Bs, the turret lathe to producing only As and split the time of the milling machines so that they spend 6 mins per hour producing Bs and the rest producing As, the total output per hour rises from 77 As and Bs to 86 As and Bs.

The key concept here is that each machine should be preferentially assigned to producing the part for which it is relatively most efficient. The relative efficiency of producing As/Bs of the three machines was milling machine $= \frac{1}{2}$, turret lathes $= \frac{2}{3}$, and automatic lathe $=\frac{3}{8}$. Clearly the turret lathe is relatively most efficient at producing As, the automatic lath relatively most efficient at producing Bs and the milling machine stands in between. Thus the automatic lathe is set to produce only Bs, the turret lathes to make only As and the time of the milling machines is split so as to ensure that an equal number of each product is turned out.

The decision process is shown diagrammatically in Figure 1. The key to the construction of the diagram, and to the decision algorithm is to rank the machines in order of their relative productivities. If one does this, one obtains a convex polygon whose line segments

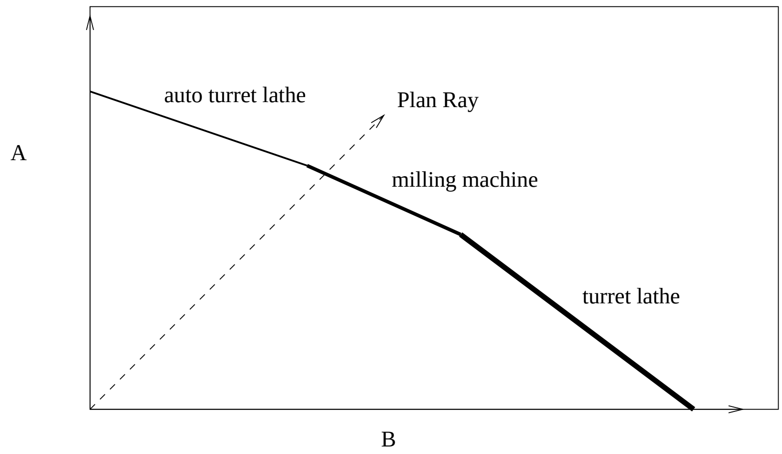

FIGURE 1. Kantorovich's example as a diagram. The plan ray is the locus all points where the output of As equals the output of Bs. The production possibility frontier is made of straight line segments whose slopes represent the relative productivities of the various machines for the two products. As a whole these make a polygon. The plan objective is best met where the plan ray intersects the boundary of this polygon.

represent the different machines. The slopes of the line segments are the relative productivities of the machines. One starts out on the left with the machine that is relatively best at producing Bs, then move through the machines in descending order of relative productivity. Because relative productivity is monotonically decreasing one is guaranteed that the boundary will be convex. One then computes the intersection of the 45 degree line representing equal output of As and Bs with the boundary of this polygon. This intersection point is the optimal way of meeting the plan. The term linear programming stems from the fact that the production functions are represented by straight lines in the case of 2 products, planes for 3 products, and for the general higher dimensional case by linear functions. That is to say, functions in which variables only appear raised to the power 1.

The slope of the boundary where the plan ray intersects was called by Kantorovich the resolving ratio. Any machine whose slope is less than this should be assigned to produce Bs any machine whose slope is greater, should be assigned to produce As.

When there are only two products being considered, the method is easy and lends itself to diagrammatic representation. But it can handle problems of higher dimensions, involving 3 or more products. In these cases we can not use graphical solutions, but Kantorovich provided an algorithmic by which by which the resolving ratios for different pairs of outputs could be arrived at by successive approximations. Kantorovich's work was unknown outside of the USSR until the late 50s and prior to that Dantzig had independently developed a similar algorithm for solving linear programming problems, the so called simplex method [Dantzig and Wolfe(1961)]. This has subsequently been incorporated into freely available software tools[5]. These packages allow you to enter the problem as a set of linear equations or linear inequalities which they then solve.

[5]For example `lp_solve` and `GLPK`.

Algorithm 1 Kantorovich's example as equations input to `lp_solve`..

```
A;
m1<=3;
m2<=3;
m3<=1;
A-B=0;
m1-0.1 x1a - 0.05 x1b=0;
m2-0.05 x2a - 0.033333 x2b=0;
m3- 0.033333 x3a - 0.0125 x3b=0;
x1a+x2a+x3a - A=0;
x1b+x2b+x3b -B =0;
int A;
```

In the West, linear programming was used to optimise the use of production facilities operating within a capitalist market. This meant that the objective function that was maximised was not a fixed mix of outputs, in Kantorovich's first example equal numbers of parts A and B, but the money that would be obtained from selling the output: price A $\times$number of As + price B $\times$number of Bs. Manuals and textbooks produced in association with Western linear programming software assumes this sort of objective. However, as we shall see, one can readily formulate Kantorovich's problem using this sort of software by adding additional equations. We shall now show how you can use the package `lp_solve` to reproduce Kantorovich's solution to his problem.

The program requires that you input an expression to be maximised or minimised followed by a sequence of equations or inequalities. In Algorithm 1 we give Kantorovich's problem in the format that `lp_solve` requires. In this example we use the following variables:

variable	meaning
A	number of units of A produced
B	number of units of N produced
m1	number of milling machines used
m2	number of turret lathes used
m3	number of automatic turret lathes used
xij	number of units of j produced on machine i

Thus x1a means the output of As on milling machines.

The first line of input is the objective function to be maximised. We give this as A, meaning maximise the output of A's. The following lines give the constraints to which the maximisation process is to be subjected.

`A-B=0`

This is another way of writing that A=B, or that equal quantities of A and B must be produced.

`m1<=3`

This means that the number of milling machines used must be less than or equal to 3. The characters '<' '=' are used because $\leq$ is not available on computer keyboards. Similar constraints are provided for the other machines.

`m1-0.1x1a-0.05x1b=0`

This specifies $m1 = 0.1x1a + 0.05x1b = \frac{1}{10}x1a + \frac{1}{20}x1b$ or in words, that allocating a milling machine to produce an A uses $\frac{1}{10}$ of a milling machine hour, and that allocating a milling machine to produce a unit of B uses $\frac{1}{20}$ of a milling machine hour. We provide similar production equations for the other machines.

`x1a+x2a+x3a - A=0`

This says that the total output of A is equal to the sum of the outputs of A from each of the machines. We provide a similar equation defining the output of B.

Note that all equations have to be provided with variables and constants on the left and a constant on the right. One can readily re-arrange the equations in this form. The last line specifies that the number of units of A produced should be an integer. When the equations are input to lp_solve it produces the answer:

```
Value of objective function: 86
Actual values of the variables:
A           86
B           86
x1a          26
x1b          6
x2a          60
x2b          0
x3a          0
x3b          80
m1           2.9
m2           3
m3           1
```

which exactly reproduces Kantorovich's own solution (Table 2) arrived at using his algorithm.

2.2. **Generalising Kantorovich's approach.** In his first example Kantorovich deals with a very simple problem, producing two goods in equal proportions using a small set of machines. He was aware, even in 1939 that the potential applications of mathematical planning were much wider. We will look at two issues that he considered which are important for the more general application of the method.

(1) Producing outputs in a definite ratio rather than in strictly equal quantities.
(2) Taking into account consumption of raw materials and other inputs.

Suppose that instead of wanting to produce one unit of A for every unit of B, as might be the case if we were matching car engines to car bodies, we want to produce 4 units of A for every unit of B, as would be the case if we were matching wheels to car engines. Can Kantorovich's method deal with this as well. Consider Figure 1 again. In that the plan ray is shown at an angle of 45° a slope of 1 to 1. If we drew the plan ray at a slope of 4 to 1, the intersection with the production frontier would provide the solution. Since this geometric approach only works for two products, let us consider the algebraic implications.

You should now be convinced that it is possible to solve Kantorovich's original problem[6] by algebraic means. In Algorithm 1 we specified that $A-B=0$ or in other words $A=B$, if one wanted 4 units of A for every B we would have to specify $A=4B$ or, expressing it in the standard form used in linear optimisation, $A-4B=0$. Suppose A stands for engines, B stands for wheels. If we now say wheels come in packs of 4, then we can repose the problem in terms of producing equal numbers of packs of wheels and engines. Introduce a new variable $\beta=4B$ to stand for packs of wheels, and rewrite the equations in terms of β and we can return to an equation specifying the output mix in the form $A-\beta=0$, which we know to be soluble.

How do we deal with consumption of raw materials or intermediate products?

In our previous example we had variables like `x1b` which stood for the output of product B on machine 1. This was always a positive quantity. Suppose that there is a third good to be considered - electricity, and that each machine consumes electricity at different rate depending on what it is turning out. Call electricity C and introduce new variables `x1ac`, `x1bc` etc referring to how much electricity is consumed by machine 1 producing outputs A and B. Then add equations specifying how much electricity is consumed by each machine doing each task, and the model will specify the total amount of electricity consumed.

We now know how to :

(1) Use Kantorovich's approach to specify that outputs must be produced in a definite ratios.
(2) Use it to take into account consumption of raw materials and other inputs.

If we can do these two tasks, we can in principle perform *in-natura* calculations for an entire planned economy. Given a final output bundle of consumer and investment goods to maximise and given our current resources, a system of linear equations and inequalities can be solved to yield the structure of the plan. From simple beginnings, optimising the output of plywood on different machines, Kantorovich had come up with a mathematical approach which could be extended to the problem of optimising the operation of the economy as a whole.

2.3. **A second example.** Let us consider a more complicated example, where we have to draw up a plan for a simple economy. We imagine an economy that produces three outputs : energy, food, and machines. The production uses labour, wind and river power, and two types of land: fertile valley land, and poorer highlands. If we build dams to tap hydro power, some fertile land is flooded. Wind power on the other hand, can be produced on hilly land without compromising its use for agriculture. We want to draw up a plan that will make the most rational use of our scarce resources of people, rivers and land.

In order to plan rationally, we must know what the composition of the final output is to be - Kantorovich's ray. For simplicity we will assume that final consumption is to be made up of food and energy, and that we want to consume these in the ratio 3 units of food

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[6] Actually this was his "problem A"

TABLE 3. Variables in the example economy

e	total energy output
e_c	household energy consumption
f	food
v	valleys
w	windmills
m	machines
d	dams
u	undamed valleys
h	highland
f_h	food produced on high land
f_v	food produced in valleys

per unit of energy. We also need to provide equations relating to the productivities of our various technologies and the total resources available to us.

Valleys are more fertile. When we grow food in a valleys, each valley requires 10,000 workers and 1000 machines and 20,000 units of energy to produce 50,000 units of food. If we grow food on high land, then each area of high land produces only 20,000 units of food using 10,000 workers, 800 machines and 10,000 units of energy.

Electricity can be produced in two ways. A dam produces 60000 units of energy, using one valley and 100 workers and 80 machines. A windmill produces 500 units of electricity, using 4 workers and 6 machines, but the land on which it is sited can still be used for farming.

We will assume that machine production uses 20 units of electricity and 10 workers per machine produced.

Finally we are constrained by the total workforce, which we shall assume to be 104,000 people.

Tables 3 and 4 show how to express the constraints on the economy and the plan in equational form. If we feed these into `lp_solve` we obtain the plan shown in Table 5. The equation solver shows that the plan targets can best be met by building no dams, generating all electricity using 541 windmills, and devoting the river valleys to agriculture.

It also shows how labour should be best allocated between activities: 40000 people should be employed in agriculture in the valleys, 109 people should work as farmers in the highlands, 2164 people should work on energy production, and 61727 people should work building machines.

The results that we have obtained were by no means obvious at the outset. It was not initially clear that it would be better to use all the river valleys for agriculture rather than building dams on some of them. In fact, whether dams or windmills are preferred turns out to depend on the whole system, not just on their individual rates of producing electricity. We can illustrate this by considering what happens if we cut the labour supply in half to 52000 people?

If we feed this constraint in to the system of equations we find the optimal use of resources has changed. The plan now involves 1 dam and 159 windmills. Cut the working

TABLE 4. Resource constraints and productivities in our example economy

final output mix	$f = 3e_c$
number of valleys	$v = 4$
dams use valleys	$v - u = d$
valley food output	$f_v = 50000u$
valley farm labour	$l_v = 10000u$
valley energy use	$e_v = 20000u$
valley farm machines	$m_v = 1000u$
highland food prod	$f_h = 20000h$
highland farm labour	$l_h = 10000h$
highland energy use	$e_h = 10000h$
highland farm machines	$m_h = 800h$
energy production	$e = 500w + 60000d$
energy workers	$l_e = 100d + 4w$
machines in energy prod	$m_e = 80d + 6w$
workers making machines	$l_m = 10m$
energy used to make machines	$e_m = 20m$
energy consumption	$e_m + e_v + e_h + e_c \leq e$
machine use	$m_e + m_h + m_v \leq m$
total food prod	$f = f_h + f_v$
workforce	$l_m + l_e + l_v + l_h \leq 104000$

TABLE 5. Economic plan for the example economy using lp_solve

d (dams)	0
e	270500
f	200218
h	0.0108889
m	6172.71
u	4
v	4
w (windmills)	541
e_c	66739.3
e_h	108.889
e_m	123454
e_v	80000
f_h	217.778
f_v	200000
l_e	2164
l_h	108.889
l_m	61727.1
l_v	40000
m_e	2164
m_h	8.71111
m_v	4000

population slightly further, down to 50000 people and the optimal plan involves flooding two valleys with dams and building only 23 windmills. Why?

As the population is reduced, there are no longer enough people available to both farm the valleys and produce agricultural machinery. Under these circumstances the higher fertility of lowland valleys is of no importance, it is better to use one or more of them to generate electricity. By applying Kantorovich's approach it becomes possible for a socialist plan to do two things that von Mises had believed impossible:

(1) It allows the plan to take into account natural resource constraints - in this case the shortage of land in river valleys which can be put to alternative uses.
(2) It allows rational choices to be made between different technologies - in this case between windmills and hydro power and between lowland and highland agriculture.

Contrary to what von Mises claimed, the whole calculation can be done in physical units without any recourse to money or to prices.

3. Valuation

The core of Mises's argument relates to the use of prices to arrive at a rational use of intermediate or capital goods. Mises argues that, in practice, only money prices will do for this, but concedes that, in principle, other systems of valuation, such as labour values would also be applicable. Kantorovich too, was very concerned with the problem of relative valuation[Kantorovich(1965)], and developed what he called *objectively determined valuations* (ODV). These valuations differed from prices, since a price involves an exchange of commodities for money between two owners. In the USSR all factories and all products were owned by the state. When products moved from one factory to another, there was no sale or purchase involved. The ODVs were purely notional numbers, used in economic calculations, not selling prices.

He considered a situation where planners have to deal with several different types of factories ($A..E$) each able to produce products 1 and 2, and where the intended ratio of output of product 1 and 2 are fixed in the plan. Each class of factory $A..E$ has different relative productivities for the two products.

He next looked at the apparent profitability of producing products 1 and 2 under different relative valuations. Under some schemes of relative price, all factories would find product 1 to be unprofitable relative to product 2, under other the reverse would occur. Intermediate price schemes would allow both products to be produced, with some classes of factories specialising on 1 and others on 2. He gives the example of childrens clothing as something which, under the administratively determined valuations then used in the USSR, were unprofitable to produce, and unless factories were specifically instructed to ignore local profitability, too few children's clothes would be made.

He asks if there exists a relative valuation structure which would allow factories to concentrate on the most valuable output, and at the same time meet the specified plan targets and arrives at certain conclusions:

(1) That among the very large number of possible plans there is always an optimal one which maximises output of plan goals with current resources.
(2) That in the optimal plan there exists a set *objectively determined valuations* (ODV) of goods which will ensure that each factory
    (a) produces the output which will contribute most to maximising the plan goals

(b) simultaneously finds the the output which contributes to maximising plan targets is the most profitable one to select

(3) With arbitrary valuations which differ from ODV, these conditions can not be met, and profit maximising factories will not specialise in a way that meets plan goals optimally.

It is important to understand that his ODVs are valuations that apply only for a plan which optimally meets a specific plan target. Kantorovichs procedure for arriving at an optimal plan involved successive adjustments to the ODVs and factory specialisation until both the appropriate mix of goods is reached, and at the same time each factory is producing its most profitable good. He actually gave several different mathematical procedures for arriving at such a plan and system of ODVs.

Although Kantorovich asserts that labour is ultimately the only source of value, his ODV's are short term valuations and differ from the classical labour theory of value, which gave valuations in terms of the long term labour reproduction costs of goods - including the reproduction costs of capital goods. Kantorovich, in constrast, is concerned with valuations which should apply with the current stock of means of production and labour resources. For example, he considers the situation of giving a valuation to electric power relative to labour. Instead of valuing it in terms of the labour required to produce electricity, he first assumes that the total electrical power available is fixed - ie, powerstations operating at full capacity, and then works out how many person hours of labour is saved by using an additional kilowatt hour of electricity. The assumption is made that in order to arrive at this objective valuation of electricity in terms of labour

(1) The plan targets must be met
(2) The plan must be optimal

Kantorovich's insistence on considering short term, very material constraints - so many megawatts of power, such and such a number of cutting machines, etc, gives his work an intensely practical and pragmatic character, quite different from that of most theoretical economists.

Why is Kantorovich so concerned with valuations and profitability?

There seem to be two reasons. We should first note that by profit maximising Kantorovich actually meant maximising the value of output. This must be understood in the context of Soviet practice where mines and factories were given incentives to overfulfill plan targets. If the output was a single good - say coal, the target could be specified in tons. But if the factory produced several goods, then the target had to be set in terms of x roubles worth of a mix of goods. With the 'wrong' price structure, plants would attempt to maximise the production of the goods which were of the highest value, ignoring those of lower value, with the result that the aggregate supply of all goods was often not in the proportions that the planners intended. This practice of setting plan targets in money terms reflected the limited ability of GOSPLAN to specify detailed targets in kind as described by [Nove(1983)].

The second reason relates to his particular algorithm for solving linear programming problems which used an iterative adjustments to initial ODVs until an optimal plan is achieved.

These two aspects seem intimately linked in his presentation, but the presuppositions about the incentives to factories are not brought to the fore.

With computer algorithms, the process of solving a linear program becomes a 'black box'. The user need not concern themselves with details such as the method of calculation - whether it uses Kantorovich's approach Danzig's or Karmarkar's, except insofar as this affects the size of problem that can be handled, as we discuss in section 4. With computer packages, ODVs would no longer be needed for computing a plan, but would they still be needed for specifying targets to factories?

This depends on the information processing capacity of the planning system. If it were capable of specifying fully disaggregated plans, then it could in principle just place orders with factories for specific quantities of each good. In these circumstances, the factories could not cheat by producing more of high value items and less of low value ones. Indeed, the very information that would be required to compute Kantorovich's ODVs, would have been sufficient for GOSPLAN to specify disaggregated orders in kind for the products that would have had valuations attached.

There remains another level at which valuations would have been useful - when product designs were being drawn up at a local level. If a refrigerator designer was deciding on what components to use in a planned new model, she would need some way of telling which components would, from a social standpoint, have been the most economical, which implies a system of valuations. However it is not clear that the full apparattus of ODVs would be either necessary or appropriate here. ODV's correspond to a system of marginal cost, rather than average cost pricing. They reflect current marginal costs with the immediately current constraints on production. The use of such marginal costing was criticised by other Soviet economists[Grossman(1963), Menshikov(2006)]. It is not clear, in retrospect, that they would have been more appropriate than a system of average cost valuation if one was projecting ahead a year or so. Indeed, given the stochastic properties of prices in a real capitalist economy[Farjoun and Machover(1983)], it is doubtful that, with the exception of certain constrained products like oil, the difference between average and marginal costs is significant in the west.

4. COMPLEXITY

Linear programming, originally pioneered by Kantorovich, provides an answer in principle to von Mises claim that rational economic calculation is impossible without money. But this is an answer only in principle. Linear programming would only be a practical solution to the problem if it were possible, in practice, to solve the equations required in a socialist plan. This in turn requires the existence of a practical algorithm for solving them, and sufficient computational resources to implement the algorithm. Kantorovich, in an appendix to [Kantorovich(1960)], gave a practical algorithm, to be executed by paper and pencil mathematics. The algorithm was sufficiently tractable for these techniques to be used to solve practical problems of a modest scale. When tackling larger problems he advised the use of approximative techniques like aggregating similar production processes and treating them as a single composite process. Whilst Kantorovich's algorithm uses his ODV's, which he earlier called *resolving multipliers*, subsequent algorithms for linear programming do not, so the ODVs should not be considered as fundamental to the field.

Since the pioneering work on linear programming in the 30s, computing has been transformed from something done by human 'computors' to something done by electronic ones. The speed at which calculations can be done has increased many billion-fold. It is now possible to use software packages to solve huge systems of linear equations. But are computers powerful enough to be used to plan an entire economy?

In a large economy like the former USSR there were probably several million distinct types of industrial products, ranging from the various sorts of screws, washers and types of electronic components to large final products like ships and airliners. Although there was great enthusiasm for Kantorovich's methods in the USSR during the 60s, the scale of the economy was to great for his techniques to be used for detailed planning with the then available computer technology. Instead they were used either in optimising particular production plants, or, in drawing up aggregated sectoral plans for the economy as a whole. How has the situation changed today, given that the power of computers has continued to grow at an exponential rate since the fall of the USSR?

4.1. **Complexity classes.** To answer this one needs to be able to quantify the complexity of a planning task and compare it with available computing resources. Measuring complexity is a branch of algorithmics. Algorithms are classified into complexity classes. For instance, computing the average of a list of n numbers is said to be of complexity class n, because the number of simple arithmetic operations required will be proportional to n. This complexity class is termed linear, as the algorithms execution time on a computer grows linearly with the number of items.

A bit more complex than linear algorithms are the log-linear ones. It turns out that one can sort a list of n numbers into ascending order using $n\log(n)$ basic arithmetic operations. Problems which are either linear or log-linear are reckoned to be very easy to solve on computers.

Next in difficulty come the polynomial problems where the number of basic arithmetic steps grows as some polynomial function of the size of the input data. If an algorithm had a running time that was proportional to n^2 or to $n^3$for some size of input data n then it would be of polynomial complexity. In algorithmics, polynomial problems are regarded as being tractable, since, with computers able to do billions of operations per second, such problems can be solved for quite large values of n. For example multiplication is a task that grows polynomially with the number of digits in the numbers. If you want to multiply 17 by 32 you have to carry out the basic steps $2\times 7 = 15$, $2\times 10 = 20$, $30\times 7 = 210$, $30\times 10 = 300$ and then add up the partial products. The number of multiplication steps will grow as n^2, where n is the number of digits in the numbers.

Beyond the polynomial problems comes the class of NP or non-deterministic polynomial problems. These are problems which, were you to take them to Oracle at Delphi, and were the priestess to give you an answer, you could check whether her answer was correct in polynomial time. Suppose you had a 100 digit number x and asked the priestess what its prime factors were, and she replied with one 47 digit number and one 53 digit number. You could take this on trust, or bearing in mind the many tales of those mislead by the Divine Oracle, you might decide to check if her answer was correct. If she were right, then multiplying the two numbers she gave you should yield x. This multiplication would take you of the order of $47 \times 53 = 2491$ basic operations, which is roughly $\frac{1}{4}n^2$ in terms of the length of the original number you gave the priestess. This shows that we can check the validity of purported prime factors in polynomial time.

Sadly, the Oracle at Delphi has long fallen from use, and we, lacking that divine guidance once available to the Ancients, must find prime factors by mundane means. A mundane and deterministic procedure is to test all $y \in 2..\sqrt{x}$ to see if $\frac{x}{y}$ is a whole number. The first such y is a prime factor. The drawback is the vast number of tests that must be performed. For 100 digit numbers we would have to test all y in the range $2..10^{50}$ to be sure of finding a prime factor if one existed. The number of tests to be performed grows as $10^{\frac{n}{2}}$, in other words the number of tests grows exponentially with n. This problem, and others in the class of exponential problems, is assumed to be computationally intractable, since the number of possibilities to be checked grows so rapidly that it rapidly exhausts the power of even the swiftest computer. Indeed so hard is the task that certain cryptographic protocols[Rivest et al.(1978)Rivest, Shamir, and Adleman] rely on large prime factors being practically impossible to discover.

4.2. **Complexity class of economic planning.** After that short introduction to the idea of complexity classes, let us apply these ideas to economic planning. To what complexity class does linear programming belong?

> For a long time it was not known whether or not linear programs belonged to a non-polynomial class called "hard" (such as the one the traveling salesman problem belongs to) or to an "easy" polynomial class (like the one that the shortest path problem belongs to). In 1970, Victor Klee and George Minty created an example that showed that the classical simplex algorithm would require an exponential number of steps to solve a worst-case linear program. In 1978, the Russian mathematician L. G. Khachian developed a polynomial-time algorithm for solving linear programs. It is an interior method using ellipsoids inscribed in the feasible region. He proved that the computing time is guaranteed to be less that a polynomial expression in the dimensions of the problem and the number of digits of input data. Although polynomial, the bound he established turned out to be too high for his algorithm to be used to solve practical problems.
>
> Karmarkar's algorithm [Karmarkar(1984)] was an important improvement on the theoretical result of Khachian that a showed how linear program can be solved in polynomial time. Moreover his algorithm turned

> out to be one which could be used to solve practical linear programs. (*Dantzig* [Dantzig(2002)])

Modern linear programming packages tend to combine Dantzig's simplex method with the more recent interior point methods. This allows the most modern implementations to solve programming problems involving up to one billion variables[Gondzio and Grothey(2006), Gondzio and Grothey(2005)]. For such huge problems large parallel supercomputers with over a thousand processor chips are used. But even with much more modest 4 CPU computers, linear programming problems in the million variable class were being solved in half an hour using interior point methods[7].

These advances in linear programming algorithms and in computer technology mean that linear programming could now be applied to detailed planning at the whole economy level, rather than just at an aggregate level.

5. Deriving the plan ray

Kantorovich assumed that the plan had a given target to optimise in the form of a particular mix of goods: the plan ray. This reflected the social reality for those engaged in managing Soviet industry, in that they were given a mix of products to produce by GOSPLAN. The planning authorities themselves however, needed to specify what this ultimate output mix would be. In the early phases of Soviet planning, when Kantorovich wrote his original paper, the goals set by the planners were primarily directed at achieving rapid industrialisation and building up a defence base against the threat of invasion. The planning process was successful in achieving these goals. But in an already industrialised country, in times of peace, the meeting of current social needs becomes the first priority and so plan vector has to be pointed in that direction. A criticism commonly levelled at the Soviet-type economies—and not only by their Western detractors—is that they were unresponsive to consumer demand. It is therefore important to our general argument to demonstrate that a planned economy can be responsive to the changing pattern of consumer preferences—that the shortages, queues and surpluses of unwanted goods of which we hear so much are not an inherent feature of socialist planning. The economists Dickinson and Lange, writing just prior to Kantorovich, outlined a practical mechanism by which this could be done [Lange(1938), Dickinson(1933)].

They proposed that the state wholesale sector should operate on a break-even basis with flexible prices. Wholesale managers would set market clearing prices for the products on sale as consumer goods. These wholesale prices would then act as a guide to the plan authorities, telling them whether to increase or decrease production of particular lines of product. If prices were high, then that line of product would have its output increased, otherwise its planned output would be reduced.

The basic idea is clear, the same principle that adjusts production of consumer goods in a capitalist economy was to be employed. But this then raises the problem of how one determines that a price is high or low. High or low relative to what?

[7] See[Bienstock(2002)] chap 4. The Harmony Algorithm for constructing plans, given in [Cottrell and Cockshott(1992)], is an instance of the class of algorithm discussed by Bienstock.

What would be the basis of valuation used?

After incorrectly rejecting the possibility of planning in kind, Mises had considered the possibility that the socialist planners might be able to make use of an 'objectively recognizable unity of value', i.e., some measurable property of goods, in performing their economic calculations. The only candidate Mises could see for such a unit is labour content, as in the theories of value of Ricardo and Marx. The latter had proposed that workers be paid in labour tokens and that goods be priced similarly[Marx(1970)]. Mises ended up rejecting labour as a value unit; he had two relevant arguments, each purporting to show that labour content cannot provide an adequate measure of the cost of production. These arguments concern the neglect of natural resource costs implicit in the use of labour values, and the inhomogeneity of labour. Mises' critique of labour values is very brief and sketchy. Two pages or so of substantive argument appear in [von Mises(1935)] and are reproduced in [von Mises(1949)]. This doubtless reflects the fact that although Marx and Engels had laid great stress on planning as an allocation of labour time, this conception had been more or less abandoned by English speaking socialist economists by the late 30s. Neither Lange nor Dickinson relied on the classical theory of value in their arguments. Writing in 1930, Appel[Appel et al.(1990 (1930))Appel, Baker, Deutschlands, and Kommunisten] had laid great stress on the relevance of the labour theory of value for socialist economics, but his ideas were largely ignored. More recent writers have again laid emphasis on Marx's theory of value as a guide to socialist planning[Dieterich(2002), Peters(1996), Peters(2000), Cottrell and Cockshott(1992)].

The basic principle of in these schemes can be stated quite simply. All consumer goods are marked with their labour values, i.e. the total amount of social labour which is required to produce them, both directly and indirectly. But aside from this, the actual prices (in labour tokens) of consumer goods will be set, as far as possible, at market-clearing levels. Suppose a particular item requires 10 hours of labour to produce. It will then be marked with a labour value of 10 hours, but if an excess demand for the item emerges when it is priced at 10 labour tokens, the price will be raised so as to (approximately) eliminate the excess demand. Suppose this price happens to be 12 labour tokens. This product then has a ratio of market-clearing price to labour-value of 12/10, or 1.20. The planners record this ratio for each consumer good. We would expect the ratio to vary from product to product, sometimes around 1.0, sometimes above (if the product is in strong demand), and sometimes below (if the product is relatively unpopular). The planners then follow this rule: Increase the target output of goods with a ratio in excess of 1.0, and reduce the target for goods with a ratio less than 1.0.

The point is that these ratios provide a measure of the effectiveness of social labour in meeting consumers' needs (production of 'use-value', in Marx's terminology) across the different industries. If a product has a ratio of market-clearing price to labour-value above 1.0, this indicates that people are willing to spend more labour tokens on the item (i.e. work more hours to acquire it) than the labour time required to produce it. But this in turn indicates that the labour devoted to producing this product is of above-average 'social effectiveness'. Conversely, if the market-clearing price falls below the labour-value, that tells us that consumers do not 'value' the product at its full labour content: labour devoted to this good is of below-average effectiveness. Parity, or a ratio of 1.0, is an equilibrium condition: in this case consumers 'value' the product, in terms of their own labour time, at just what it costs society to produce it.

The feasibility of using labour time for expressing prices depends on being able to calculate it. This might seem a daunting task, but it actually involves solving a similar, though somewhat easier, set of linear equations to those required when one draws up a consistent plan. The task is thus computationally tractable on the grounds explained earlier.

Mises' objected that "the ... defect in calculation in terms of labour is the ignoring of the different qualities of labour" (1935: 114). Mises notes Marx's claim that skilled labour counts as a multiple of, and hence may be reduced to, 'simple labour', but argues that there is no way to effect this reduction short of the comparison of the products of different labours in the process of market exchange. Wage differentials might appear to offer a solution, but the equalizing process in this case "is a result of market transactions and not its antecedent." Mises assumes that the socialist society will operate an egalitarian incomes policy, so that market-determined wage rates will not be available as a guide to calculation. The conclusion is then that "calculation in terms of labour would have to set up an arbitrary proportion for the substitution of complex by simple labour, which excludes its employment for purposes of economic administration" (1935: 115).

True, labour is not homogeneous, but there is no warrant for the claim that the reduction factor for complex labour has to be arbitrary under socialism. There are two possible approaches:

(1) Skilled labour may be treated in the same way that Marx treats the means of production in Capital, namely as a produced input which 'transfers' embodied labour to its product over time. Given the labour time required to produce skills and a depreciation horizon for those skills, one may calculate an implied 'rate of transfer' of the labour time embodied in the skills. If we call this rate, for skill i, r_i , then labour of this type should be counted as a multiple (1+r_i) of simple labour, for the purpose of 'costing' its products. An iterative procedure is needed: first calculate the transfer rates as if all inputs were simple labour, then use those first-round transfer rates to re-evaluate the skilled labour inputs, on this basis recompute the transfer rates, and so on, until convergence is reached.

(2) Alternatively one may use the approach advocated by Kantorovich[Kantorovich(1965)](page 64..66) where he shows that skilled labour of different grades can be assigned ODVs on the basis of their different productivities.

Which method is usesed woud depend on the timescale of the calculation. If one wants short term answers to the relative valuations of different labours, then Kantorovich's approach is relevant. For longer term considerations, within the time scale that newly trained staff can be brought up to speed, then the first alternative would be appropriate.

6. CONCLUSION

The Soviet mathematical school founded by Kantorovich and the Austrian school exemplified by Mises and Hayek took radically different positions on the feasibility of socialist economic calculation. To a large extent they ignored one another. The Austrian school largely concentrated on criticising Western trained socialist economists like Lange and the Soviet school appears to have ignored Mises completely. Even when the key participants met, the issue was not raised. Menshikov writes:

> It is interesting that in the account of his trip to Sweden for receiving the Nobel Prize, Kantorovich mentions an informal reception with the participation of several American economists – Nobel Prize laureates – including Hayek, Leontief, and Samuelson. But, apparently, neither at this reception, nor during other meetings, this issue was never raised. In January 1976, when I worked in USA as the Director of the United Nation Projections and Perspective Studies Branch, I was asked to present L. V. Kantorovich as a new Nobel Prize laureate at the annual meeting of the American Economic Association in Atlantic City. Of course, I put the emphasis on the economic discovery of the laureate. In the discussion, none of the audience, which included T. Koopmans and L. Klein, a future Nobel Prize laureate, mentioned the question of actual Kantorovich's answer to a part of Hayek's argumentation.[Menshikov(2006)]

With the political demise of the USSR, the Austrian school have tended to assume that Mises arguments have been vindicated, but theoretical economic arguments are not finally resolved by politics. Poltical fashions change. Socialism, from being politically unpopular in Europe the 1990s, has, since then, been making substantial inroads on another continent. No, one has to bring economic arguments head to head in their own terms. Kantorovich, an absent participant in the Western debate on socialist calculation, is worth paying attention to.

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