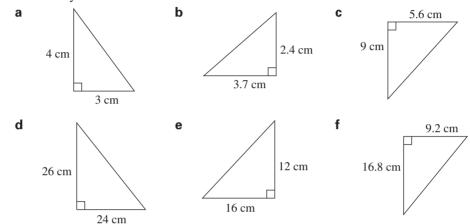
Geometry: Pythagoras and trigonometry

5.1 Pythagoras' theorem

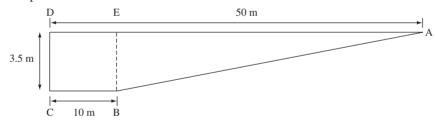
HOMEWORK 5A

1 In each of the following triangles, find the hypotenuse, rounding off to a suitable degree of accuracy.



2 This diagram shows the cross-section of a swimming pool 50 m long. It is 3.5 m deep at the deep end. The deepest part of the pool is 10 m long. It is not drawn to scale.

- **a** Calculate the length of the sloping bottom of the pool AB.
- **b** The pool is 20 m wide. What is its volume?



AU 3

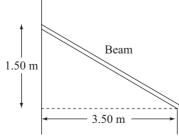
Three of these lengths could form the sides of a right-angled triangle: $7.5 \text{ cm} \quad 10 \text{ cm} \quad 12.5 \text{ cm} \quad 15 \text{ cm}$

Which one would not be used? Give a reason for your answer.

(FM 4

A beam of wood is needed to support a sloping roof, as shown. The beam spans a horizontal distance of 3.50 m and the difference between the bottom and the top is 1.50 m.

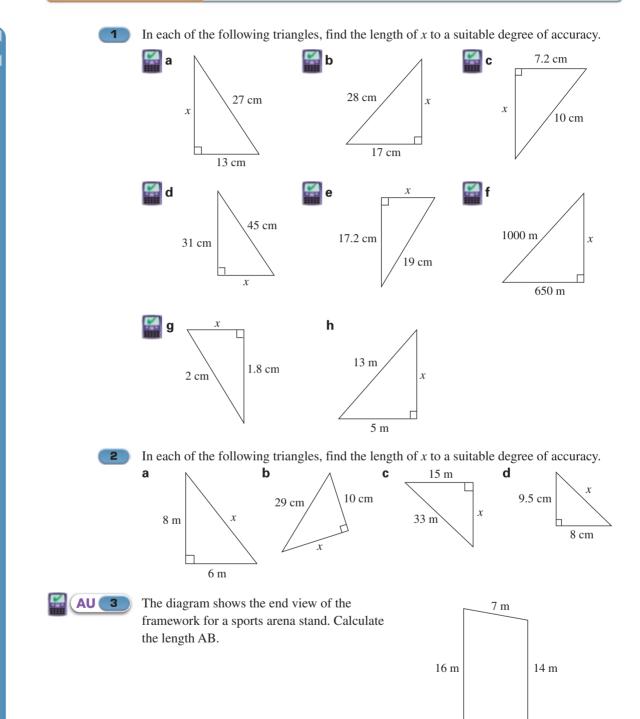
33



A builder has a beam that is 4 m long. Is it long enough?

5.2 Finding a shorter side

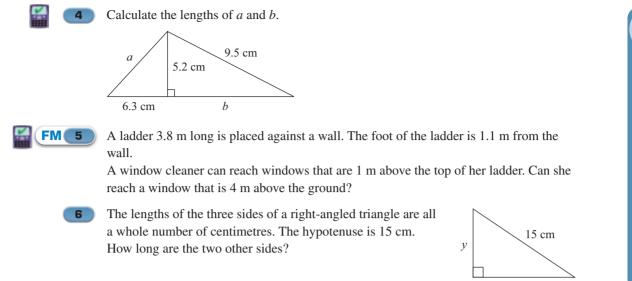
HOMEWORK 5B



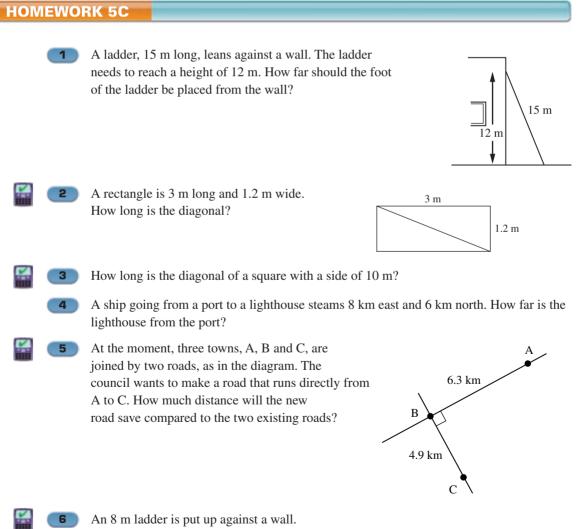
В

А

34 UNIT 3



5.3 Applying Pythagoras' theorem in real situations

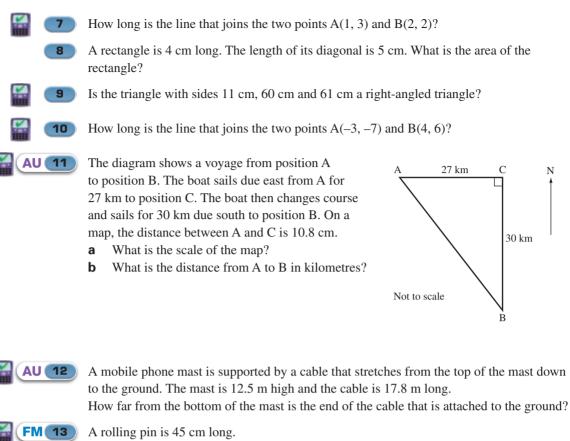


a How far up the wall will it reach when the foot of the ladder is 1 m away from the wall?

b When it reaches 7 m up the wall, how far is the foot of the ladder away from the wall?

35

UNIT 3

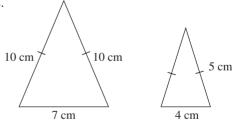


Will it fit inside a kitchen drawer which is internally 40 cm long and 33 cm wide? Justify your answer.

HOMEWORK 5D



Calculate the area of these isosceles triangles.



2 Calculate the area of an isosceles triangle with sides of 10 cm, 10 cm and 8 cm.

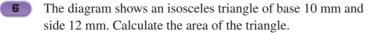


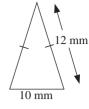
4

5

Calculate the area of an equilateral triangle with sides of 10 cm each.

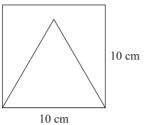
- **a** Calculate the area of an equilateral triangle with sides of 20 cm each.
- **b** Explain why the answer to **4a** is not twice that of Question **3**.
- An isosceles triangle has sides of 6 cm and 8 cm.
- **a** Sketch the two isosceles triangles that fit this data.
- **b** Which of the two triangles has the greater area?







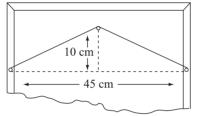
The diagram shows an equilateral triangle drawn inside a square with sides of 10 cm each.



What percentage of the square is outside the triangle?



A picture is hanging on a string secured to two points at the side of the frame.



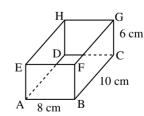
The string is initially 45 cm long. When the picture is hung the string stretches as shown. By how much does the string stretch?

5.4 Pythagoras' theorem in three dimensions

HOMEWORK 5E 41 cm Is the triangle with sides of 9 cm, 40 cm and 41 cm a 9 cm right-angled triangle? 40 cm 2

A box measures 6 cm by 8 cm by 10 cm.

- Calculate the lengths of
- i AC ii BG iii BE
- b Calculate the diagonal distance BH.



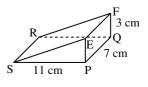


а

A garage is 5 m long, 5 m wide and 2 m high. Can a 7 m long pole be stored in it?

Spike, a spider, is at the corner S of the wedge shown in the diagram. Fred, a fly, is at the corner F of the same wedge.

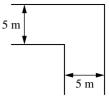
- Calculate the two distances Spike would have to а travel to get to Fred if she used the edges of the wedge.
- Calculate the distance Spike would have to travel across b the face of the wedge to get directly to Fred.

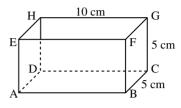


5 A corridor is 5 m wide and turns through a right angle, as in the diagram. What is the longest pole that can be carried along the corridor horizontally? If the corridor is 3 m high, what is the longest pole that can be carried along in any direction?

For the box shown on the right, find the lengths of:

- **a** DG
- **b** HA
- c DB
- **d** AG







FM

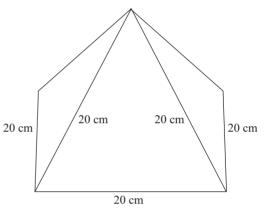
8

6

A cube has a side of 15 cm.

Calculate the distance between two vertically opposite corners.

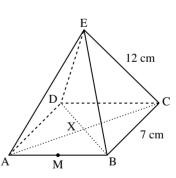
A small sculpture is made from four equilateral triangles of copper sheet stuck together to make a pyramid.



The triangles have a side of 20 cm. How high is the pyramid?

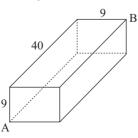
9 The diagram shows a square-based pyramid with base length 7 cm and sloping edges 12 cm. M is the mid-point of the side AB, X is the mid-point of the base, and E is directly above X.

- **a** Calculate the length of the diagonal AC.
- **b** Calculate EX, the height of the pyramid.
- **c** Using triangle ABE, calculate the length EM.





Use the answer to Question 1 to find the length of the diagonal AB of the cuboid 9 cm by 9 cm by 40 cm.





5.5 Trigonometric ratios

HOMEWOR	RK 5F
	In these questions, give any answers involving angles to the nearest degree.
1	Find these values, rounding off your answers to 3 significant figures. a $\sin 52^{\circ}$ b $\sin 46^{\circ}$ c $\sin 76.3^{\circ}$ d $\sin 90^{\circ}$
2	Find these values, rounding off your answers to 3 significant figures. a $\cos 52^{\circ}$ b $\cos 46^{\circ}$ c $\cos 76.3^{\circ}$ d $\cos 90^{\circ}$
3	a Calculate $(\sin 52^\circ)^2 + (\cos 52^\circ)^2$ b Calculate $(\sin 46^\circ)^2 + (\cos 46^\circ)^2$ c Calculate $(\sin 76.3^\circ)^2 + (\cos 76.3^\circ)^2$ d Calculate $(\sin 90^\circ)^2 + (\cos 90^\circ)^2$ e What do you notice about your answers?
4	Use your calculator to work out the value of: a $\tan 52^{\circ}$ b $\tan 46^{\circ}$ c $\tan 76.3^{\circ}$ d $\tan 0^{\circ}$
5	Use your calculator to work out the value of: a $\sin 52^\circ \div \cos 52^\circ$ b $\sin 46^\circ \div \cos 46^\circ$ c $\sin 76.3^\circ \div \cos 76.3^\circ$ d $\sin 0^\circ \div \cos 0^\circ$ e What connects your answers with the answers to Question 4 ?
6	Use your calculator to work out the value of: a $6 \sin 55^{\circ}$ b $7 \cos 45^{\circ}$ c $13 \sin 67^{\circ}$ d $20 \tan 38^{\circ}$
7	Use your calculator to work out the value of: a $\frac{6}{\sin 55^{\circ}}$ b $\frac{7}{\cos 45^{\circ}}$ c $\frac{13}{\sin 67^{\circ}}$ d $\frac{20}{\tan 38^{\circ}}$
8	Using the following triangle, calculate sin, cos, and tan for the angle marked <i>x</i> . Leave your answers as fractions.
AUS	You are given that $\sin x = \frac{5}{\sqrt{34}}$. Work out the value of $\tan x$.

5.6 Calculating angles

HOMEWORK 5G

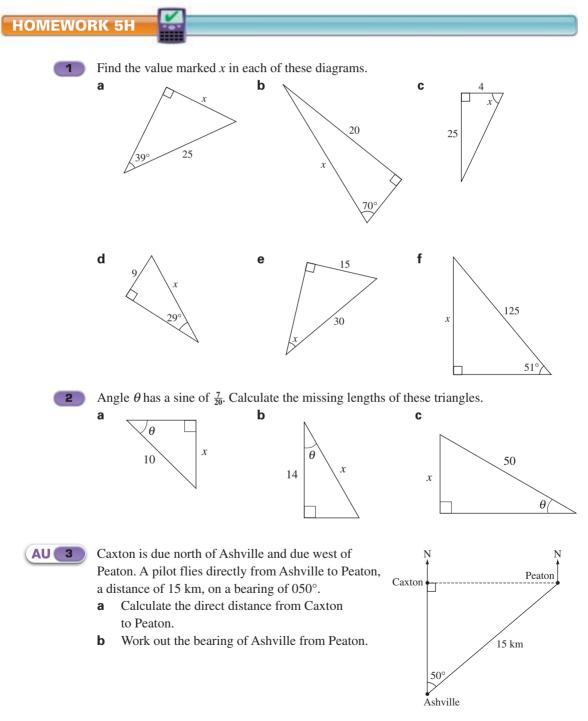
Use your calculator to find the answers to the following to one decimal place.

1	1 What angles have sines of:							
	а	0.4	b	0.707	С	0.879	d	0.6666666666666666
2	Wł	nat angles hav	e co	sines of:				
	а	0.4	b	0.707	С	0.879	d	0.333333333333333
3	Wł	nat angles hav	e the	e following tar	ngen	ts?		
_	а	0.4	b	1.24	С	0.875	d	2.625
4	Wł	hat angles hav	e the	e following sin	nes?			
	а	3 ÷ 8	b	1 ÷ 3	С	$3 \div 10$	d	5 ÷ 8

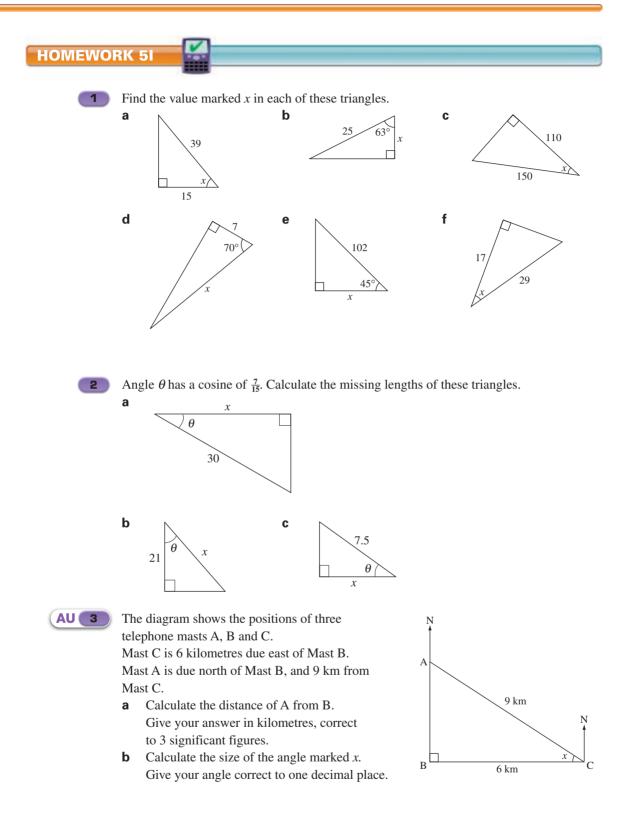
5	What angles have the following cosines?						
	a 3÷	b	1 ÷ 3	C	3 ÷ 10	d	5 ÷ 8
6		ngles have the	e following tai 3 ÷ 2	C	ts? 5 ÷ 7	d	19 ÷ 5

7 If $\sin 54^\circ = 0.809$ to 3 decimal places, what angle has a cosine of 0.809?

5.7 Using the sine and cosine functions

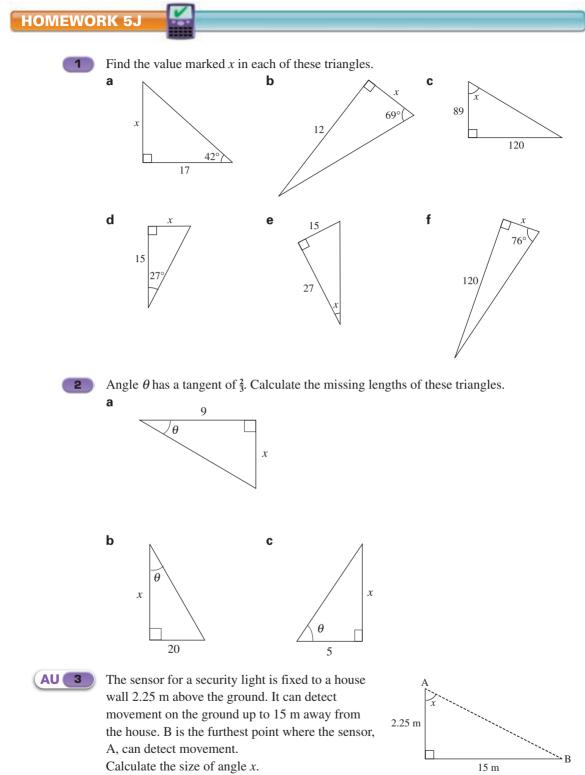


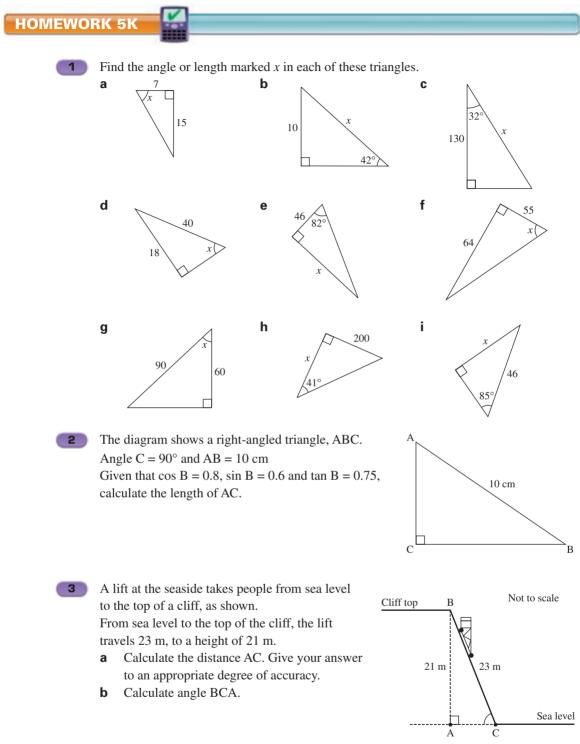
40 UNIT 3



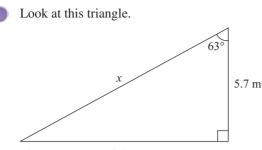
UNIT 3 41

5.8 Using the tangent function





5.9 Which ratio to use



Find the length of side *x*.

5.10 Solving problems using trigonometry 1

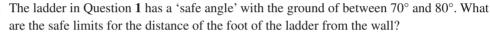
HOMEWORK 5L

In these questions, give any answers involving angles to the nearest degree.

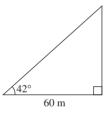


A ladder, 8 m long, rests against a wall. The foot of the ladder is 2.7 m from the base of the wall. What angle does the ladder make with the ground?

FM 2



Angela walks 60 m from the base of a block of flats and then measures the angle from the ground to the top of the flats to be 42° as shown in the diagram. How high is the block of flats?





A slide is at an angle of 46° to the horizontal. The slide is 7 m long. How high is the top of the slide above the ground?



5

FM 6

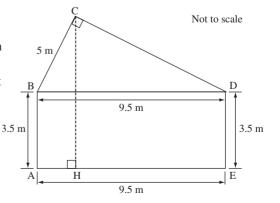
AU (7

Use trigonometry to calculate the angle that the diagonal makes with the long side of a rectangle 9 cm by 5 cm.

Drumsbury Town Council wants to put up a flag pole outside the town hall. The diagram shows the end view of the town hall building.

Regulations state that the flag pole must not be more than half the height of the building.

What is the maximum height that the flag pole can be?

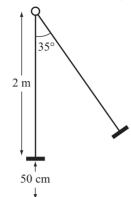


A road rises steadily at an angle of 6°. A lorry travels 300 m along the road. What is the increase in height?

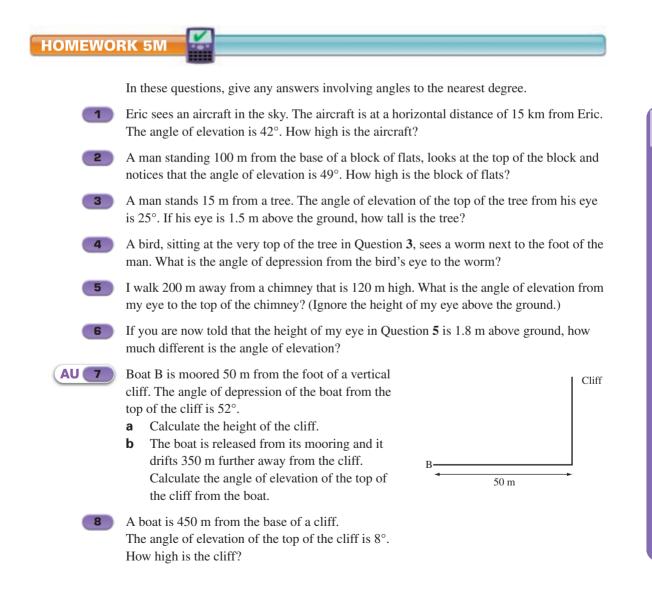
UNIT 3



A swing at rest is 50 cm above the ground and 2 m below the point of suspension. When a child is on the swing, the angle with the vertical can be as large as 35°.



The child's father thinks that she might then be dangerously far from the ground. Can you tell him exactly how far his daughter will be above the ground?





9 To find the height of a tree, Sacha tries to measure the angle of elevation of the top from a point 40 m away.

He finds it difficult to measure the angle accurately, but thinks it is between 30° and 35° . What can you tell him about the height of the tree?

5.11 Solving problems using trigonometry 2

HOMEWORK 5N	

FM

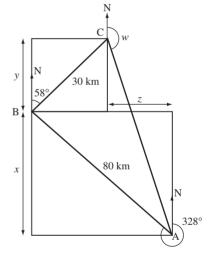
- A ship sails for 85 km on a bearing of 067°.
 - **a** How far east has it travelled?
 - **b** How far north has the ship sailed?
- **2** Rotherham is 11 miles south of Barnsley and 2 miles west of Barnsley. What is the bearing of:
 - a Barnsley from Rotherham b Rotherham from Barnsley?

3 A plane sets off from airport A and flies due east for 100 km, then turns to fly due south for 80 km before landing at an airport B. What is the bearing of airport B from airport A?

Mountain A is due east of a walker. Mountain B is due south of the walker. The guidebook says that mountain A is 5 km from mountain B, on a bearing of 038°. How far is the walker from mountain B?

5 The diagram shows the relative distances and bearings of three ships A, B and C.

- **a** How far north of A is B? (Distance *x* on diagram.)
- **b** How far north of B is C? (Distance *y* on diagram.)
- **c** How far west of A is C? (Distance *z* on diagram.)
- **d** What is the bearing of A from C? (Angle *w* on diagram.)

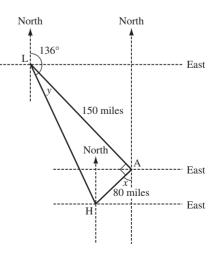




FM 4

An aeroplane is flying from Leeds (L) to London Heathrow (H). It flies 150 miles on a bearing 136° to point A. It then turns through 90° and flies the final 80 miles to H.

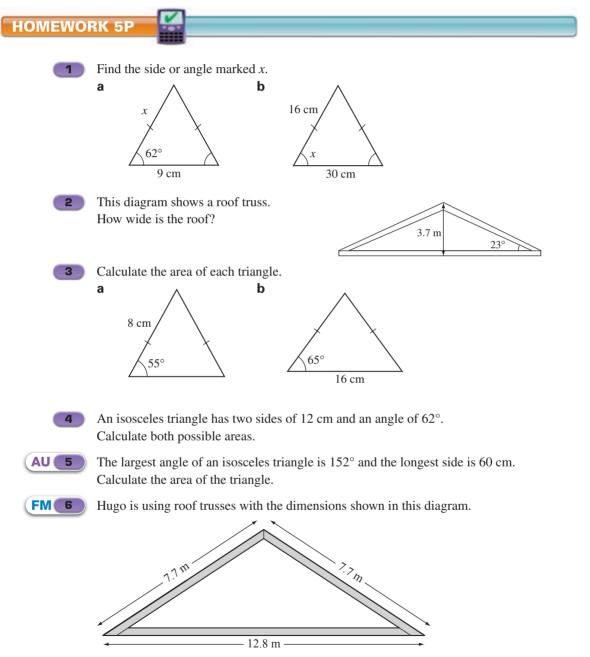
- **a** i Show clearly why the angle marked x is equal to 46° .
 - **ii** Give the bearing of H from A.
- **b** Use Pythagoras' theorem to calculate the distance LH.
- **c i** Calculate the size of the angle marked *y*.
 - ii Work out the bearing of L from H.



AU **7** A plane flies 200 km on a bearing of 124° and then 150 km on a bearing of 053°. How far east from its starting point has it travelled?

FM 8

Large boats are supposed to stay at least 300 m from the shore near a particular beach.
Don notices a large boat that is due north from where he is sitting on the beach.
He walks 100 m to the east and uses a compass to find that the bearing of the boat is 340°.
Is the boat breaking the rules?



What is the angle of slope of the roof?

Functional Maths Activity Access ramps

Building regulations in the UK state how steep wheelchair ramps used to access buildings are allowed to be.

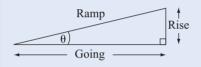
Going	Maximum gradient	Maximum rise
10 m	1:20	500 mm
5 m	1:15	333 mm
2 m	1:12	166 mm

Below are some definitions of the jargon used. The diagram that follows illustrates how they are used in practice.

Going: The horizontal length.

Gradient: The tangent of the angle the ramp makes with the horizontal. Here it is written as a ratio.

Rise: The change in height from one end of the ramp to the other.



Gradient = tan θ

You can add to the table, using different lengths for the going. Try adding the maximum gradient and maximum rise for goings of 4 m and 9 m, following the number patterns in the table.

A builder has asked you to explain a few things that are puzzling him.

- **a** Is there a connection between the numbers in the three columns?
- **b** What is the difference between the maximum angles for a 2 m going and a 10 m going?
- **c** The builder has been asked to install an access ramp to an old building. The rise required is 400 mm. In order not to exceed the available space, he wants to build a ramp with a 7 m going. Is this permitted, according to the regulations? Explain how you would decide.

5 Geometry: Pythagoras' theorem and trigonometry

5.1 Pythagoras' theorem

HOMEWORK 5A

- 1
 a
 5 cm
 b
 4.41 cm
 c
 10.6 cm

 d
 35.4 cm
 e
 20 cm
 f
 19.2 cm
- **2 a** 40.15 m **b** 2100 m³
 - 100 m³
- **3** 15 cm, because $7.5^2 + 10^2 = 12.5^2$
- **4** 3.81 metres, so the beam is long enough

5.2 Finding a shorter side

HOMEWORK 5B

- 1
 a
 23.7 cm
 b
 22.3 cm
 c
 6.9 cm

 d
 32.6 cm
 e
 8.1 cm
 f
 760 m

 g
 0.87 cm
 h
 12 m
 2
 a
 10 m
 b
 27.2 cm
 c
 29.4 m

 d
 12.4 cm
 5
 5
 5
 5
 5
 5
- **3** 6.7 m
- **4** a = 8.2 cm b = 8.0 cm
- 5 No, the ladder will only reach 3.6 metres
- 6 3 cm and 5 cm

5.3 Applying Pythagoras' theorem in real-life situations

HOMEWORK 5C

- **1** 9 m
- 2 3.23 m
- **3** 14.14 m
- 4 10 km
- 5 3.22 km 6 a 7.9 m

b 3.9 m

- **7** $\sqrt{2}$
- **8** 12 cm²
- 9 Yes, $61^2 = 60^2 + 11^2$
- 10 14.76 units
- 11 a 1 cm represents 2.5 km
- 12 12.7 metres
- **13** The diagonal of the drawer is $\sqrt{(40^2 + 33^2)} = 51.8$ cm, so it will fit in the drawer if it is put in at an angle.

HOMEWORK 5D

- **1** 32.8 cm², 9.17 cm²
- **2** 36.7 cm²
- **3** 43.3 cm²

b 40.4 km

```
4 a 173.2 cm<sup>2</sup>
```

- **b** Only the lengths have doubled; the area has quadrupled.
- **5 a** Student's sketches
 - **b** 8, 8, 6 has area 22.25 cm^2 and 6, 6, 8 has 17.9 cm^2
- **6** 54.5 mm²
- **7** 56.7%
- 8 49 cm or 49.2 cm

5.4 Pythagoras' theorem in three dimensions

HOMEWORK 5E

1	Yes							
2	a i	AC = 12.8 cm	ii	BG = 11.7 cm	iii	BE = 10.0 cm		
		H = 14.1 cm						
	Yes							
		1 cm and 18.4 cm	b	13.4 cm				
		m and 14.5 m						
			b	HA = 7.1 cm	С	DB = 11.2 cm		
		G = 12.2 cm						
	26 cn							
	14.1 (_					
	a A 42 cn		b	EX = 10.9 cm	С	EM = 11.5 cm		
10	42 01	1						
5.	5 Trig	onometric ratios						
нс	MEW	ORK 5F						
1	a 0.	788	b	0.719	С	0.972		
	d 1							
2	a 0.	616	b	0.695	С	0.237		
	d 0							
3	a 1		b	1	С	1		
	d 1		е	All 1				
4	a 1.	280	b	1.036	С	4.102		
	d 0							
5	a 1.	280	b	1.036	С	4.102		
	d 0		е	same				
6	a 4.	915	b	4.950	С	11.967		
	d 1							
7	a 7.		b	9.899	С	14.123		
	d 2	5.60						
8	8 Sin $x = \frac{5}{13}$, cos $x = \frac{12}{13}$, tan $x = \frac{5}{12}$							

9 Tan $x = \frac{5}{3}$

5.6 Calculating angles

нс	HOMEWORK 5G								
1	а	23.6°	b	45.0°	С	61.5°			
	d	41.8°							
2	а	66.4°	b	45.0°	С	28.5°			
	d	70.5°							
3	а	21.8°	b	51.1°	С	41.2°			
	d	69.1°							
4	а	22.0°	b	19.5°	С	17.5°			
	d	38.7°							
5	а	68.0°	b	70.5°	С	72.5°			
	d	51.3							
6	а	20.6°	b	56.3°	С	35.5°			
	d	75.3°							
7	36	.0°							

5.7 Using the sine and cosine functions

HOMEWORK 5H

1	а	15.7	b	21.3	С	80.9°		
	d	18.6	е	30°	f	97.1		
2	а	3.5	b	14.95	С	17.5		
3	а	11.5 km	b	230°				
HOMEWORK 5I								
1	а	67.4°	b	11.3	С	42.8°		
	d	20.5	е	72.1	f	54.1°		

2	а	14	b	45	c	3.5
3	а	6.71 km	b	48.2°		

5.8 Using the tangent function

HOMEWORK 5J

1	а	15.3	b	4.6	С	53.4°
	d	7.64	е	29.1°	f	29.9
2	а	6	b	30	С	$\frac{10}{3}$

3 81.5°

5.9 Which ratio to use

HOMEWORK 5K

1	a 65.0°	b 14.9	С	153.3
	d 26.7°	e 327	f	49.3°
	g 48.2°	h 230	i	45.8
2	6 cm			
3	a 9.4 m	b 65.9°		

4 12.6 cm

5.10 Solving problems using trigonometry 1

HOMEWORK 5L

- **1** 70.3°
- 2 2.74 m to 1.39 m
- **3** 54 m
- **4** 5.04 m
- **5** 29°
- 6 3.88 m
- 7 31 metres
- 8 The swing will rise to a maximum height of 86 cm, or 36 cm above its initial height.

HOMEWORK 5M

- 1 13.5 km
- **2** 115 m
- **3** 8.5 m
- **4** 29.5° (30° to the nearest degree)
- **5** 31°
- **6** 0.4° (0° to the nearest degree)
- **7 a** 64 m **b** 9.1° (9° to the nearest degree)
- 8 63 metres
- 9 It is probably between 23 and 28 metres high.

5.11 Solving problems using trigonometry 2

HOMEWORK 5N

- **1 a** 78.2 km **b** 33.2 km
- **2 a** 10.3° **b** 190.3°
- **3** 128.7°
- 4 3.94 km
- **5 a** 67.8 km **b** 15.9 km **c** 17.0 km
 - **d** 168.6°
- **6 a i** Example of proof: Remaining angle at L between LA and the vertical is $180 136 = 44^{\circ}$ (angles on a straight line). Therefore the angle at A between LA and the vertical (North) is 44° because LA is transversal between the two North parallel lines. Therefore $x = 180 90 44 = 46^{\circ}$ (angles on a straight line).
 - ii 226°
 - **b** 170 km
 - **c** i 28.1° ii 344.1°
- 7 286 kilometres
- 8 Yes: it is only 275 metres from the shore.

HOMEWORK 5P

- **1 a** 9.59 cm **b** 20.4°
- **2** 17.4 m
- **3 a** 30.1 cm² **b** 137.2 cm²
- **4** 63.6 cm², 59.7 cm²
- **5** 224 cm²
- **6** 34°

Functional Maths Activity

Access ramps

- **a** The ratio of the maximum rise to the going is equal to the gradient.
- **b** The angles are 2.9 degrees and 4.7 degrees, a difference of 1.9 degrees.
- **c** Yes. By interpolation, the maximum gradient is 1 : 17 and the corresponding maximum rise is 412 mm, which is greater than the 400 mm he wants.