MAXIMAL RED SHIFTS OF NEUTRON STARS

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Abstract. It has been recently established that there exists a maximal red shift $z_{\max}$ for a homogeneous star of given mass M. The relationship $z_{\max}(M)$ is obtained for neutron stars in the mass range $0.71 \leqslant M/M_{\odot} \leqslant 12.06$.

1. Introduction

In a recent paper (Kovetz, 1969) it has been shown that among all homogeneous isentropic configurations of given mass M there exists one with minimal entropy $s_{\min}(M)$. Furthermore, the corresponding radius $R(M, s_{\min})$ is the lowest possible, and hence the red shift

$$Z_{\max}(M) = [1 - 2GM/c^2 R(M, s_{\min})]^{-1} - 1 \tag{1}$$

is the highest possible for this mass.

The purpose of this paper is to obtain maximal red shifts $z_{\max}(M)$ for neutron stars. For cold ($s = T = 0$) configurations results were obtained by Oppenheimer and Volkoff (1939). In Section 2 we present results for masses beginning with the limiting cold neutron star $M = 0.71\,M_{\odot}$ up to $M = 12\,M_{\odot}$. It will be seen that for higher masses the calculation can be carried out on the post-Newtonian approximation (Chandrasekhar, 1965).

2. Calculation of Maximal Red Shifts

For a neutron gas we have the formulae (Rakavy and Shaviv, 1967)

$$p = Cm_n c^2 F_1(\psi, \beta) = \tfrac{1}{3} Cm_n c^2 \int_1^{\infty} \frac{(x^2 - 1)^{3/2}\,\mathrm{d}x}{e^{\beta x - \psi} + 1}, \tag{2}$$

$$n = CF_2(\psi, \beta) = C \int_1^{\infty} \frac{x(x^2 - 1)^{1/2}\,\mathrm{d}x}{e^{\beta x - \psi} + 1}, \tag{3}$$

$$e = Cm_n c^2 F_3(\psi, \beta) = Cm_n c^2 \int_1^{\infty} \frac{x^2(x^2 - 1)^{1/2}\,\mathrm{d}x}{e^{\beta x - \psi} + 1}, \tag{4}$$

for the pressure, number density and energy density, respectively, where m_n denotes the neutron mass, $C = (m_n c/\hbar)^3/\pi^2$, $\beta = m_n c^2/kT$ and $\psi = \mu/kT$, with μ the chemical potential; while k, c and $\hbar$ are, respectively, Bolzmann's constant, the velocity of light and Planck's constant divided by 2π. We note the definition

$$\psi = \varepsilon_f + \beta \tag{5}$$

of the Fermi parameter (or degeneracy parameter) ε_f.

The entropy per neutron s is given by

$$s/k = \beta (F_1 + F_3)/F_2 - \psi . \tag{6}$$

To these we add the corresponding quantities for the radiation field

$$p_r = C m_n c^2 \frac{\pi^4}{45} \beta^{-4}, \tag{7}$$

$$e_r = 3 p_r, \tag{8}$$

$$s_r/k = \frac{4\pi^4 \beta^{-3}}{45 F_2}. \tag{9}$$

To obtain an isentropic configuration we use the field equations in the form

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 e/c^2, \tag{10}$$

$$\frac{\mathrm{d}\varepsilon_F}{\mathrm{d}r} = \left(\frac{\partial \varepsilon_f}{\partial p}\right)_s \frac{\mathrm{d}p}{\mathrm{d}r} = -\left(\frac{\partial \varepsilon_f}{\partial p}\right)_s \frac{Gme}{c^2 r^2} \frac{(1 + p/e)(1 + 4\pi r^3 p/mc^2)}{1 - 2Gm/rc^2}, \tag{11}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}r} = \left(\frac{\partial \beta}{\partial p}\right)_s \frac{\mathrm{d}p}{\mathrm{d}r} = -\left(\frac{\partial \beta}{\partial p}\right)_s \frac{Gme}{c^2 r^2} \frac{(1 + p/e)(1 + 4\pi r^3 p/mc^2)}{1 - 2Gm/rc^2}. \tag{12}$$

The thermodynamic derivatives can be expressed in terms of F_1, F_2, F_3 and the further functions (Rakavy and Shaviv, 1967)

$$F_4(\psi, \beta) = \beta \frac{\partial F_2}{\partial \psi}, \tag{13}$$

$$F_5(\psi, \beta) = -\frac{\partial}{\partial \beta}(\beta F_2), \tag{14}$$

$$F_6(\psi, \beta) = -\frac{1}{\beta}\frac{\partial}{\partial \beta}(\beta^2 F_3). \tag{15}$$

In the non-degenerate ($\varepsilon_F < -4$), non-relativistic ($\beta > 32$) and extremely degenerate ($\varepsilon_F > 20$) regions the functions were calculated using the well-known asymptotic expressions* (e.g. Guess, 1966). Where these asymptotic expressions do not apply, the functions were evaluated numerically with the aid of a computer program published by Guess (1966).

With given values for ε_f and β at the center ($r = 0$), an isentropic configuration is

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