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Mathematics for All

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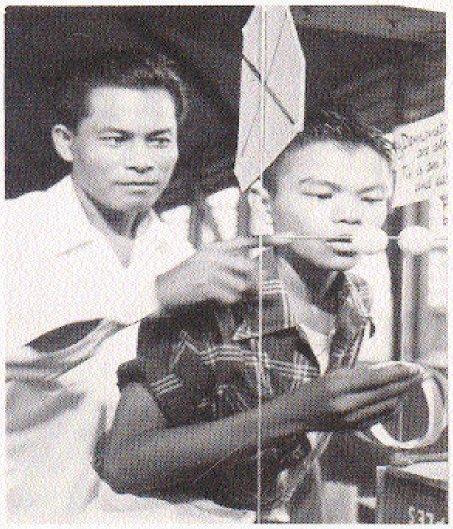

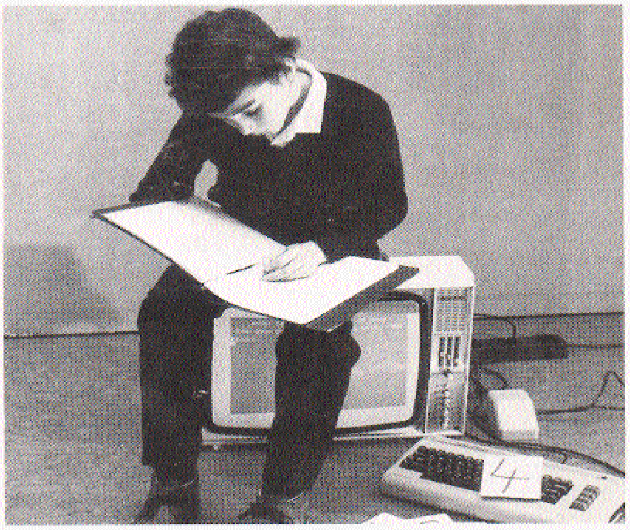

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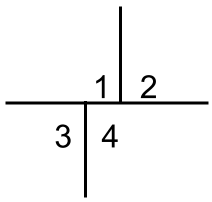

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Science and Technology Education

Document Series No. 20

Mathematics for All

Problems of cultural selectivity and unequal distribution of mathematical education and future perspectives on mathematics teaching for the majority

Report and papers presented in theme group I,
'Mathematics for All'
at the 5th International Congress on Mathematical Education,
Adelaide, August 24-29, 1984

Edited by
Peter Damerow, Mervyn E. Dunkley, Bienvenido F. Nebres and Bevan Werry

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Preface

This resource document consists of twenty-two papers prepared by authors from all regions and presented at the Fifth International Congress on Mathematical Education (ICME 5). Over 2000 mathematics educators from sixty-nine countries gathered in Adelaide, Australia, in August 1984, to discuss problems in their field. This document is one outcome. Its purpose is to continue the dialogue to assist nations in their search for a mathematics programme for all students.

Mathematics for All is the first document in mathematics education in Unesco's Science and Technology Education Document Series. This, coupled with Unesco's publications *Studies in Mathematics Education* and *New Trends in Mathematics Teaching,* was initiated to encourage an international exchange of ideas and information.

Unesco expresses its appreciation to the editors, Peter Damerow, Mervyn Dunkley, Bienvenido Nebres and Bevan Werry for their work, to the Max Planck Institute for Human Development and Education for preparing the manuscript, and to the ICME 5 Programme Committee for permitting Unesco to produce this report.

The views expressed in the text are those of the authors and not necessarily those of Unesco, the editors, or of ICME 5.

We welcome comments on the contents of this document. Please send them to: Mathematics Education Programme Specialist, Division of Science, Technical and Environmental Education, Unesco, Place de Fontenoy, 75700 Paris, France.

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Introduction:
Report on the Work of Theme Group I «Mathematics for All» at ICME 5

1. Introduction

Many factors have brought about a change in the overall situation of mathematics education. These include the move to universal elementary education in developing countries, the move to universal secondary education in industrialised countries (where there have also been growing demands for mathematical competence in an increasingly technologically and scientifically oriented world) and from the experience gained with worldwide curriculum developments such as the new mathematics movement. The tacit assumption, that what can be gained from mathematics can be gained equally in every culture and independently of the character of the school institution and the individual dispositions and the social situations of the learner, turned out to be invalid. New and urgent questions have been raised. Probably the most important ones are:

- What kind of mathematics curriculum is adequate to the needs of the majority?
- What modifications to the curriculum or alternative curricula are needed for special groups of learners?
- How should these curricula be structured?
- How could they be implemented?

A lot of work has already been done all over the world in attempts to answer these questions or to contribute to special aspects of the problem.

- ICME 4 yielded several presentations of results concerning universal basic education, the relationship of mathematics to its applications, the relation between mathematics and language, women and mathematics, and the problems of teaching mathematics to special groups of students whose needs and whose situations do not fit into the general framework of traditional mathematics education.
- The Second International Mathematics Study of the International Association for the Evaluation of Educational Achievement (IEA) dealt much more than the first one with the similarities and differences of the mathematics curriculum in different countries, and the different conditions which determine the overall outcome in mathematical achievement. The IEA collected data on both the selectivity of mathematics and the differences between countries in the way they produce yield levels of mathematical qualification. Although final reports on the Second International Mathematics Study are not yet available, preliminary analyses of the data have already produced useful results.
- In several countries national studies have been concerned with the evaluation of the mathematics education system. An important recent example is the Report of the Committee of Enquiry into the Teaching of Mathematics in Schools in England and Wales (commonly known as the Cockcroft Report) in 1982.
- Last, but not least, there are many detailed studies, projects and proposals from different countries dealing with special aspects such as:
  - teaching the disadvantaged;
  - teaching the talented;
  - teaching mathematics to non-mathematicians;
  - teaching mathematics in the context of real life situations;
  - teaching mathematics under atypical conditions, etc.

At ICME 5, papers were presented on a variety of topics related to the theme Mathematics for All. Taken as a whole, these contribute to a better understanding of the problems of teaching mathematics successfully, not only to very able students, but teaching worthwhile mathematics successfully to all in a range of diverse cultures and circumstances.

2. Summary of Papers Presented to the Theme Group

The first group of papers dealt with general aspects of the theme Mathematics for All.

Jean-Claude Martin, Rector of the Academy of Bordeaux in France, analysed in his paper, A *Necessary Renewal of Mathematics Education,* the special selectivity of mathematical education as a result of symbolism and mathematical language. The teaching of mathematics seems to have been designed to produce future mathematicians despite the fact that only a very small percentage of students reach university level. This general character of mathematical education causes avoidable, system-related failures in mathematical learning and often results in a strong aversion to mathematics. Martin proposed a general reorientation of mathematical education aiming at a mathematics which is a useful tool for the majority of students. The teaching of mathematics as a means of solving multidisciplinary problems by using modelling methods should restore student interest, show mathematics as being useful, enrich students knowledge of related subjects and so enable them better to memorise mathematical formulas and methods, encourage logical reasoning and allow more students access to a higher level of mathematics.

Bienvenido F. Nebres in his paper, *The Problem of Universal Mathematics Education in Developing Countries,* discussed the same problem of the lack of fit between the goals of mathematical education and the needs of the majority in the special circumstances of the situation in developing countries. He offered a conceptual framework for discussing the specific cultural dimensions of the problem in these countries by using the distinction between vertical and horizontal relationships, i.e. the relationships between corresponding institutions in different societies and the relationships between social or cultural institutions within the same country. The history of social and cultural institutions in developing countries is that their establishment and growth has been guided more by vertical relationships, i.e. an adaption of a similar type of institution from the mother colonial country, rather than by horizontal relationships. The result is a special lack of fit between mathematical education and the needs of the majority of the people.

There is a tremendous need for researchers in mathematics education in developing countries to look at the actual life of urban workers, rural farmers and merchants and to identify the mathematics in daily life that is needed and used by people. Then it is necessary to compare this needed mathematics with what is provided in the curriculum and to search for a better fit between the two. A cultural shift must be brought about in these countries. Mathematical educators, together with other educators and other leaders of society, should take up the need for the social and cultural institutions to be better integrated with one another and to develop together in a more organic manner than in the past.

In a joint paper *Mathematics for All: Conclusions Drawn from the Experiences of the New Mathematics Movement,* Peter Damerow of the Max Planck Institute for Human Development and Education in Berlin, West Germany, and Ian Westbury of the University of Illinois at Urbana-Champaign, United States, examined the problem of designing a mathematics curriculum which genuinely meets the diverse needs of all students in a country. They argue that, by continuing to ignore the needs of all except a small minority of students, the curricula developed within the new mathematics movement proved to be no more satisfactory than their predecessors. Traditionally, mathematics curricula were developed for an elite group of students who were expected to specialise in the subject, and to study mathematics subsequently at higher levels in a tertiary institution. As education has become increasingly universal, however, students of lesser ability, and with more modest vocational aspirations and daily life requirements, have entered the school system in greater numbers. A major problem results when these students are exposed to a curriculum designed for potential specialists. This same type of traditional curriculum has frequently been transferred to developing and third world countries, where, because of different cultural and social conditions, its inappropriateness for general mathematical education has only been compounded. So called reforms such as new mathematics did little to resolve the major problems in that they merely attempted to replace one specialist curriculum by another.

The question addressed by Damerow and Westbury is how to cater both for the elite and also for the wider group of students for whom mathematics should be grounded in real world problem solving and daily life applications. One suggestion is that the majority would achieve a mathematical «Literacy» through the use of mathematics in other subjects such as science, economics, while school mathematics would remain essentially and deliberately for specialists. This is effectively to retain the status quo. Alternatively, mathematics must be kept as a fundamental part of the school curriculum, but ways of teaching it effectively to the majority must be found. The majority of students will be users of mathematics. Damerow and Westbury concluded that a mathematics program which is truly for all must seek to overcome the subordination of elementary mathematics to higher mathematics, to overcome its preliminary, preparatory character, and to overcome its irrelevance to real life situations.

The findings of the Second International Mathematics Study (SIMS) were used by Howard Russell, Ontario Institute for Studies in Education, Canada, in his paper *Mathematics for All: SIMS Data,* to argue that mathematics is already taught to all pupils at the elementary level in many countries. At the senior secondary level, however, the prevailing pattern in most countries is for mathematics to be taught only to an elite. At the lower level, the SIMS data suggest that promotion by age, rather than by performance, does not violate the concept of mathematics for all. The SIMS data also appear to provide support for the Cockcroft hypothesis that the pace of mathematics education must be slowed if sufficient students are to be retained in mathematics courses at the higher levels for it to be accurately labelled mathematics for all. Alternatively, the content of the curriculum could be trimmed down as suggested by Damerow. Russell proposed a market-oriented rationale to construct such a core of material, particularly to meet the needs of the middle level students who will be required to use mathematics in their chosen work in the market place.

Afzal Ahmed was a member of the Committee of Inquiry into the Teaching of Mathematics in Schools in England and Wales (Cockcroft Committee), and is now the director of the Curriculum Development Project for Low Attaining Pupils in Secondary School Mathematics. In his paper, *The Foundations of Mathematics Education for All,* he discussed implications of the Cockcroft Report, published in January 1982, concerning the major issues of the theme group. He pointed out that a suitable mathematics curriculum for the majority assumes greater importance as societies in the world become more technological and sophisticated. But at the same time, the evidence of failure at learning and applying mathematics by a large proportion of the population is also growing. The Cockcroft Report proposes a Foundation List of Mathematical Topics that should form part of the mathematics syllabus for all pupils. In his discussion of the Cockcroft Report, Ahmed focussed particularly on the classroom conditions which facilitate, or inhibit the mastery of these fundamental topics.

In a paper on *Universal Mathematics Education,* Achmad Arifin from the Bandung Institute of Technology in Indonesia, described how community participation should be raised in carrying out universal mathematics education through looking at the aspect of interaction within and between social and cultural institutions. He asked the questions which parts of mathematics can function as a developer of an individual's intelligence and how should those parts that have been chosen be presented? Any program to answer these questions has to take into account three components of interaction. Firstly, depending on its quality, social structure through interaction can contribute to the improvement of peoples' abilities, especially by making them appreciate mathematics. Secondly, a special form of social interaction, which he called positive interaction, can motivate mathematics learning and create opportunities to learn. Thirdly, school interaction itself can inspire, stimulate, and direct learning activities. In developing countries, local mathematicians in particular are able to understand their cultural conditions, the needs, the challenges and the wishes of their developing nation. Taking into account the three components of interaction, they have the ability and the opportunity to spread and share their knowledge and to translate and utilise the development of mathematics in universal mathematics education for their nation.

In many countries, there is one mathematics syllabus for each year of the education system. Andrew J. C. Begg, in his paper, *Alternative Mathematics Programs,* questioned this practice and argued for the introduction of alternative mathematics programs which will meet the varied needs of all students in a range of circumstances and with a range of individual aspirations. All such courses should contribute towards general educational aims such as the development of self-respect, concern for others, and the urge to enquire. Thus, mathematics courses should provide an opportunity to develop skills of communication, responsibility, criticism, and cooperation. Such an approach has implications for the way in which students are organised in mathematics classes; for the scheduling of mathematics classes; for the choice of teaching and learning methods; for the extent to which emphasis is placed on cooperation as against competition; for the use of group methods of teaching; and for the provision that should be made for students from diverse cultural groups. In this way, mathematics programs for all students should assist not only the achievement of mathematical objectives, but also the attainment of personal, vocational and humanistic aims in education. By matching mathematics programs to the needs of students, the development of the self-esteem of every student becomes central in the mathematics curriculum.

The second group of papers was concerned with particular concerns related to Mathematics for All in industrialised countries.

In their paper, *Arithmetic Pedagogy at the Beginning of the School System in Japan,* Genichi Matsubara and Zennosuke Kusumoto traced the introduction of the teaching of western arithmetic to Japan in the late nineteenth century. At a time when universal elementary education was only just approaching reality in Japan, the government declared a policy of adopting western-style arithmetic in order to enable the country to compete more successfully internationally. This move faced obstacles in its implementation because of the traditional use of the abacus and the widespread lack of familiarity with the Hindu-Arabic notation. Further, in a developing national system of education, teachers were in short supply and little attention could be given to teaching methods in the training courses. The paper emphasised the need to make such changes slowly and to take into account the situation of those closely involved with the changes if they are to be successful in modifying the curriculum for mathematics for all.

The extent to which the mathematics learnt at school is retained and used in later life is the subject of research reported in a paper by Takashi Izushi and Akira Yamashita of Fukuoka University, Japan, entitled *On the Value of Mathematical Education Retained by the Social Members of Japan in General.* A study in 1955 was concerned with people who had learnt their mathematics before the period in Japan in which mathematics teaching was focussed on daily life experience and before compulsory education was extended to secondary schools. Although it was found that most people retained the mathematics skills and knowledge well, rather fewer claimed that this material was useful in their work. A second more limited study in 1982 confirmed these general findings in relation to geometry. It showed, broadly speaking, that younger people tended to use their school mathematics more directly while older people relied more on common sense. The study covered a further aspect, the application of the attitudes of deductive thinking derived from the learning of geometry. The thinking and reasoning powers inculcated by this approach were not forgotten and were claimed to be useful in daily life, but not in work. Izushi and Yamashita conclude that the inclusion of an element of formal mathematical discipline in the curriculum is supported by Japanese society.

Another attempt to create a modern course in advanced mathematics which is also worthwhile for those students who don’t intend to proceed to university was reported by Ulla Kürstein Jensen from Denmark in her paper titled *Upper Secondary Mathematics for All? An Evolution and a Draft.* The increase from about 5% in former years to about 40So in 1983 of an age cohort completing upper secondary education with at least some mathematics brought about an evolution toward a curriculum concentrating on useful mathematics and applications in daily life and mathematical modelling. This evolution led to the draft of a new curriculum which will be tested under school conditions, beginning in autumn 1984. The origin of this development is based on new regulations for mathematical education for the upper secondary school in the year 1961. It was influenced by the new mathematics ideas and designed to serve the needs of the small proportion of the students passing through upper secondary education at that time, but soon had to be modified for the rapidly increasing number of students in the following years. So the mathematics teaching, particularly for students in the language stream of the school system, was more and more influenced by ideas and teaching materials of a further education program which was much more related to usefulness for a broad part of the population than the usual upper secondary mathematics courses. In 1981, this development was legitimated by new regulations and, by that time, even mathematics teaching in classes concentrating on mathematics and physics became more and more influenced by the tendency to put more emphasis on applications leading ultimately to the draft of the new unified curriculum which is now going to be put into practice.

The central topic of a paper entitled *Fight against School Failure in Mathematics,* presented by Josette Adda from the Université Paris 7, was an analysis of social selective functions of mathematical education. She reported statistical data showing the successive elimination of pupils from the «normal way at each decision stage of the school system until only 16% of the 17 year age cohort remain whereas all others have been put backward or relegated to special types of classes. These eliminations hit selectively socioculturally disadvantaged families. Research studies, particularly at the Université Paris 7, have been undertaken to find out why mathematics teaching as it is practised today is not neutral but produces a correlation between school failure in mathematics and the sociocultural environment. They indicate the existence of parasitic sources of misunderstanding increasing the difficulties inherent in mathematics, e. g. embodiments of mathematics in pseudo-concrete situations which are difficult to understand for many pupils. On the other hand, it had been found that children failing at school are nevertheless able to perform authentic mathematical activities and to master logical operations on abstract objects.

Two papers were based on the work of the EQUALS program in the United States. This is an intervention program developed in response to a concern about the high dropout rate from mathematics courses, particularly in the case of women and minority students. The program aims to develop students' awareness of the importance of mathematics to their future work, to increase their confidence and competence in doing mathematics, and to encourage their persistence in mathematics.

In the first of these papers, *EQUALS: An Inservice Program to Promote the Participation of Underrepresented Students in Mathematics,* Sherry Fraser described the way in which the program has assisted teachers to become more aware of the problem and the likely consequences for individual students of cutting themselves off from a mathematical education. By working with teachers and providing them with learning materials and methods, with strategies for problem solving in a range of mathematical topics, together with the competence and confidence to use these, EQUALS has facilitated and encouraged a transfer of concern to the classroom and attracted and retained greater numbers of underrepresented students in mathematics classes. Since 1977, 10,000 educators have participated in the program.

Although the main focus of activity in the EQUALS program has been on working with teachers and administrators, needs expressed by these educators for materials to involve parents in their children's mathematical education led to the establishment of *Family Math.* Virginia Thompson described how this project has developed a curriculum for short courses where parents and their children can meet weekly to learn mathematical activities together to do at home. This work reinforces and complements the school mathematics program. Although the activities are suitable for all students, a major focus has been to ensure that underrepresented students, primarily females and minorities, are helped to increase their enjoyment of mathematics. The project serves to reinforce the aims of the EQUALS program.

The move over the past ten years or so towards applicable, real world and daily life mathematics in the Netherlands, inspired by the work of Freudenthal, was described by Jan de Lange Jzn. of OW and OC, Utrecht, in his *paper Mathematics for All is No Mathematics at All.* Textbooks have been published for primary and lower secondary schools which reflect this view of mathematics, and research shows that the reaction of teachers and students has been very favourable. De Lange illustrated the vital role played by applications and modelling in a newly-introduced curriculum for pre-university students. Many teachers apparently view the applications-oriented approach to mathematics very differently from the traditional mathematics content. The ultimate outcome, de Lange suggested, may be that science and general subjects will absorb the daily life use of mathematics and consequently this type of mathematics might disappear from the mathematics curriculum. That is, the ultimate for all students as far as mathematics is concerned could in reality become no mathematics as such.

Roland Stowasser from the Technical University of West Berlin proposed in his paper, *Problem Oriented Mathematics Can be Taught to All,* to use examples from the history of mathematics to overcome certain difficulties arising from courses based on a single closed system, which increase mathematical complexity but do not equally increase the applicability to open problems. He stated that mathematics for all does not necessarily have to be directly useful, but it has to meet two criteria: The mathematical ideas have to be simple, and on the other hand, they have to be powerful. He illustrated these criteria through a historical example. Regiomantus formulated the problem to find the point from which a walking person sees a given length high up above him (e. g. the minute hand of a clock if the person walks in the same plane as the face of the clock) subtending the largest possible angle. The solution with ruler and compasses in the framework of Euclidean geometry is somewhat tricky. But according to Stowasser the teaching of elementary geometry should not be restricted to Greek tricks. For problem solving he advocated free use of possible tools, and the solution of the problem is very simple if trial and error methods are allowed. So the solution of the historical problem represents the simple but powerful idea of approximation.

What are the characteristics of a mathematics program suitable for all students, and do any such programs exist? These questions were addressed by Allan Podbelsek of the United States in his paper, *Realization of a Mathematics Program for All.* Podbelsek listed a number of criteria for such a program covering not only content knowledge and skills but also attitudes towards, and beliefs about, mathematics and the process skills involved in its use. Mathematics must be seen to be a unified, integrated subject, rather than a set of individual, isolated topics. The Comprehensive School Mathematics Program

(CSMP) developed over several years in the United States for elementary (K-6) level classes is found to meet these criteria successfully in almost every respect. Practical problems involved in the introduction of such a program as CSMP to a school were discussed by Podbelsek. These problems centred on the provision of adequate teacher training for those concerned, meeting the cost of materials, securing the support of parents and the local community, and ensuring that administrative staff were aware of the goals of the program.

Those translating mathematical, scientific or technical material should have a basic knowledge of mathematics to do their job satisfactorily, yet because of their language background they are not likely to have studied mathematics to any great extent at school. This is the experience which led Manfred Klika, of the Hochschule Hildesheim in West Germany, to a consideration of the nature and adequacy of present school mathematics programs in his paper *Mathematics for Translators Specialised in Scientific Texts - A Case Study on Teaching Mathematics to Non-Mathematicians.* Conventional school programs, he claimed, do not prepare students to comprehend and make sense of mathematical ideas and terminology. The solution is to construct the mathematics curriculum around fundamental ideas. Two perspectives on this notion are offered- major anathematising models (e. g. mathematical concepts, principles, techniques, etc.) and field-specified strategies suitable for problem solving in mathematics (e.g. approximate methods, simulation, transformation strategies, etc.). A curriculum based on such fundamental ideas would result in more meaningful learning and thus a more positive attitude to the subject. A course based on this approach has been established at the Hochschule Hildesheim within the program for training specialist translators for work in technical fields.

The major concern of the preceding contributions to the topic " Mathematics for All " were problems of designing a mathematics curriculum which is adequate to the needs and the cognitive background of the majority in industrialised countries. The organising committee of the theme group was convinced that it is even more important to discuss the corresponding problems in developing countries. But it was much more difficult to get substantial contributions in this domain. To stress the importance of the development of mathematical education in developing countries, the work of the theme group terminated with a panel discussion on *Universal Mathematical Education in Developing Countries,* with short statements of major arguments by Bienvenido F. Nebres from the Philippines, Terezinha N. Carraher from Brazil, and Achmad Arifin from Indonesia, followed by the reactions of Peter Towns and Bill Barton, both from New Zealand. The discussion concentrated on the relation between micro-systems of mathematical education like curricula, textbooks and teacher training and macro-systems like economy, culture, language and general educational systems which, particularly in the developing countries, often determine what kind of developments on the level of micro-systems are possible. Bienvenido F. Nebres expressed the common conviction of the participants when he argued that, in spite of the fact that often it is impossible to get a substantial improvement of mathematics education without fundamental changes in the macro-systems of education, micro-changes are possible and are indeed a necessary condition to make people realise what has to be done to get a better fit between mathematical education and the needs of the majority. This result of the discussion highlights the importance of the papers submitted to the theme group dealing with special aspects of mathematical education in developing countries.

Three reports were given by David W. Carraher, Terezinha N. Carraher and Analucia D. Schliemann about research undertaken at the Universidade Federal de Pernambuco in Recife, Brazil. David W. Carraher prepared a paper titled *Having a Feel for Calculations* about a study investigating the uses of mathematics by young, schooled street vendors who belong to social classes characteristically failing in grade school, often because of problems in mathematics, but who often use mathematics in their jobs in the informal sector of the economy. In this study, the quality of mathematical performance was compared in the natural setting of performing calculations in the market place and in a formal setting similar to the situation in a classroom. Similar or formally identical problems appeared to be mastered significantly better in the natural setting. The reasons were discussed and it was stated that the results of the analysis strongly suggest that the errors which the street vendors make in the formal setting do not reflect a lack of understanding of arithmetical operations but rather a failing of the educational system which is out of touch with the cognitive background of its clientele. There seems to be a gulf between the intuitive understanding which the vendors display in the natural setting and the understanding which educators try to impart or develop.

Terezinha N. Carraher reported in her paper *Can Mathematics Teachers Teach Proportions?* results of a second research project. Problems involving proportionality were presented to 300 pupils attending school in Recife, in order to find out whether a child already understands proportions if it only follows correctly the routines being taught at school. The results indicate characteristic types of difficulties appearing in certain problems, some of which can be related to cognitive development. It is suggested that teachers' awareness of such difficulties may help to improve their teaching of the subject. For if mathematics is to be useful to everyone, mathematics teachers must consider carefully issues related to the transfer of knowledge acquired in the classroom to other problem solving situations.

The third paper, presented by Analucia D. Schliemann, *Mathematics Among Carpentry Apprentices: Implications for School Teaching,* highlighted the discontinuity between formal school methods of problem solving in mathematics and the informal methods used in daily life. This research study contrasted the approaches to a practical problem of quantity estimation and associated calculation taken by a group of experienced professional carpenters without extensive schooling, and a group of carpentry apprentices attending a formal school system and with at least four years of mathematics study. The results showed that apprentices approached the task as a school assignment, that their strategies were frequently meaningless and their answers absurd. On the other hand, the professional carpenters took it as a practical assignment and sought a feasible, realistic solution. Very few computational mistakes were made by either group but the apprentices appeared unable to use their formal knowledge to solve a practical problem. Schliemann concluded that problem solving should be taught in practical contexts if it is to have transferability to daily life situations out of school.

Pam Harris from the Warlpiri Bilingual School discussed in her paper, *Is Primary Mathematics Relevant to Tribal Aboriginal Communities?,* the problem that, in the remote Aboriginal communities of Australia, teachers often get the feeling that mathematics is not relevant. Several reasons can be identified. Teachers often receive negative attitudes from other people so that they go to an Aboriginal community expecting that their pupils will not be able to do mathematics. Furthermore, they observe a lack of reinforcement of mathematics in the pupils' home life. Teaching materials mostly are culturally and linguistically biased. Teachers feel discouraged because of the difficulties of teaching mathematics under these conditions. Nevertheless, Pam Harris stressed the importance of mathematics, because Aboriginal children have to get an understanding of the Second culture» of their country. They need mathematics in their everyday life, in employment, and in the conduct of community affairs. But to be successful, mathematics teaching in Aboriginal communities has to allow for and support local curriculum development. Individual schools and language groups should make their own decisions on the use of the children's own language, the inclusion of indigenous mathematical ideas, priorities of topics, and sequencing the topics to be taught.

Kathryn Crawford, from the College of Advanced Education in Canberra, presented a paper on *Bicultural Teacher Training in Mathematics lEducation for Aboriginal Trainees from Traditional Communities* in Central Australia. She described a course which forms part of the Anagu Teacher Education Program, an accredited teacher training course intended for traditionally oriented Aboriginal people currently residing in the Anagu communities who wish to take on greater teaching responsibilities in South Australian Anagu schools. The most important difference between this teacher training course and many others is that it will be carried out on site by a lecturer residing within the communities and that, from the beginning, development of the curriculum has been a cooperative venture between lecturers and educators on the one hand, and community leaders and prospective students on the other. The first group of students will begin the course in August 1984. The course is particularly designed to meet the fact that different cultures emphasise different conceptual schemes. Thus, temporal sequences and quantitative measurement are dominant themes in industrialised Western cultures but largely irrelevant in traditional Aboriginal cultures. To overcome these difficulties, the focus of the problem is redirected from the "failings" of Aboriginals and Aboriginal culture to the inappropriateness of many teaching practices for children from traditionally oriented communities. The course has been developed based on a model designed to maximise the possibility of interaction between the world view expressed by Anagu culture and that of Anglo-European culture as evidenced in school mathematics. This is achieved by placing an emphasis on the student expertise and contribution in providing information about Anagu world views as a necessary part of the course. In this community based teacher training course, it seems that it is possible for the first time to develop procedures for negotiating meanings between the two cultures.

3. Conclusions

The presentations given at the sessions of the theme group summarised above can be considered as important efforts to contribute to the great program of teaching mathematics successfully not only to a minority of selected students but teaching it successfully to all. But in spite of all these efforts it has to be admitted that the answer to. the question, What kind of mathematics curriculum is adequate to the needs of the majority?», is still an essentially open one. However, the great variety of the issues connected with this problem which were raised in the presented papers makes it at least clear that there will be no simple answer. Thus the most important results of the work of this theme group at ICME 5 may be that the problem was for the first time a central topic of an international congress on mathematical education, and that, as the contributions undoubtedly made clear, this problem will be one of the main problems of mathematical education in the next decade.

As far as the content of these contributions is concerned, the conclusion can be drawn that there are at least three very different dimensions to the problem which contribute to and affect the complex difficulties of teaching mathematics effectively to the majority:

- the influence of social and cultural conditions;
- the influence of the organisational structure of the school system;
- the influence of classroom practice and classroom interaction.

Cultural Selectivity

One of the major underlying causes of the above problem is the fact that mathematical education in the traditional sense had its origins in a specific western European cultural tradition. The canonical curriculum of «Traditional mathematics» was created in the 19th century as a study for an elite group. It was created under the conditions of a system of universal basic education which included the teaching of elementary computational skills and the ability to use these skills in daily life situations. There is a clear distinction between the aims and objectives of this basic education and the curriculum of traditional school mathematics which was aimed at formal education not primarily directed at usefulness and relevance for application and practice. This special character of the canonical mathematics school curriculum is still essentially the same today in many countries.

The transfer of the European mathematics curriculum to developing countries was closely associated with the establishment of schools for the elite by colonial administrations. Under these circumstances it seemed natural to simply copy European patterns. It is quite another problem to build a system of mass education in the Third World and embed mathematics education in both the school situation and the specific social and cultural contexts of that world.

The papers summarised above point clearly to some of the problems. Curricula exist which encourage students to develop antipathies towards mathematics; this is commonly the case in Europe. Further, such curricula have sometimes been transferred to countries where the social context lacks the culturally based consensus that is found in Europe, namely, that abstract mathematical activity is good in itself and must therefore be supported, even if it seems on the surface to be useless. It has been proposed on the one hand, that a sharp distinction should be made between applicable arithmetic in basic education and essentially pure mathematics in secondary education, and on the other hand, that mathematics should be integrated into basic technical education. This argument raises the question of the relation between mathematics and culture which may be the first problem to address when the idea of mathematics for all is raised as a basis for a program of action.

Selectivity of the School System

While the particular curricular patterns of different societies vary, the subject is still constructed in most places so that few of the students who begin the study of mathematics continue taking the subject in their last secondary years. The separation of students into groups who are tagged as mathematically able and not able is endemic. Curricula are constructed from above, starting with senior levels, and adjusted downwards. The heart of mathematics teaching is, moreover, widely seen as being centered on this curriculum for the able, and this pattern is closely related to the cultural contexts indicated above. However, we must consider the problem of conceiving, even for industrialised societies, a mathematics which is appropriate for those who will not have contact with pure mathematics after their school days. Up to now we have made most of our students sit at a table without serving them dinner. Most attempts to face the problem of a basic curriculum reduce the traditional curriculum by watering down every mathematical idea and every possible difficulty to make it feasible to teach the remaining skeleton to the majority.

There is only a limited appeal to usefulness as an argument or a rationale for curriculum building to avoid the pitfalls of this situation. Students who will not have to deal with an explicit area of pure mathematics in their adult lives but will face instead only the exploitation of the developed products of mathematical thinking (e. g. program packages), will only be enabled by mathematics instruction if they can translate the mathematical knowledge they have acquired into the terms of real-life situations which are only implicitly structured mathematically. Very little explicit mathematics is required in such situations and it is possible to survive in most situations without any substantial mathematical attainments whatsoever.

Is the only alternative to offer mathematics to a few as a subject of early specialisation and reject it as a substantial part of the core curriculum of general education? This approach would deny the significance of mathematics. To draw this kind of conclusion we would be seen to be looking backwards in order to determine educational aims for the future. The ongoing relevance of mathematics suggests that a program of mathematics for all implies the need for a higher level of attainment than has been typically produced under the conditions of traditional school mathematics — and that this is especially true for mathematics education at the level of general education. In other words, we might claim that mathematics for all has to be considered as a program to overcome the subordination of elementary mathematics to higher mathematics, to overcome its preliminary character, and to overcome its irrelevance to life situations.

Selectivity in Classroom Interaction

Some of the papers presented in this theme group support recent research studies which have suggested that it is very likely that the structure of classroom interaction creates ability differences among students which grow during the years of schooling. In searching for causes of increasing differences in mathematical aptitude, perhaps the simplest explanation rests on the assumption that such differences are due to predispositions to mathematical thinking, with the further implication that nothing can be done really to change the situation. But this explanation is too simple to be the whole truth. The understanding of elementary mathematics in the first years of primary school is based on preconditions such as the acquisition of notions of conversation of quantity which are, in their turn, embedded in exploratory activity outside the school. The genesis of general mathematical abilities is still little understood. The possibility that extra-school experience with mathematical or pre-mathematical ideas influences school learning cannot be excluded. Furthermore, papers presented to the theme group strongly suggest that the differences between intended mathematical understanding and the understanding which is embedded in normal classroom work is vast. We cannot exclude the possibility that classroom interaction in fact produces growing differences in mathematical aptitude and achievement by a system of positive feedback mechanisms which increase high achievement and further decrease low achievement.

It is clear that to talk of mathematics for all entails an intention to change general attitudes towards mathematics as a subject, to eliminate divisions between those who are motivated towards mathematics and those who are not, and to diminish variance in the achievement outcomes of mathematics teaching. This, in its turn, involves us in an analysis of social contexts, curricula and teaching. It is these forces together which create a web of pressures which, in turn, create situations where mathematics becomes one of the subjects in the secondary school in which selection of students into aptitude and ability groups is an omnipresent reality almost from the time of entry.

Part I:
Mathematics for All –
General Perspectives

A Necessary Renewal of Mathematics Education

Jean-Claude Martin

Mathematics for all must not only be accessible mathematics, but interesting mathematics for all - or for the majority.

Such a theory leads one, in the case of the teaching of mathematics in France, to raise problems of objectives and curriculum organisation, but also of methods more than of the content of the curriculum.

1. The General Characteristics of Selective Education

(i) The Fundamental Teaching of Mathematics for Mathematics Sake

Mathematics as they are known today may be considered, if not as a whole, as a system. The training of the highest level of generalist mathematicians may a priori be defined as leading to knowledge of this system.

Dividing the system of mathematics into parts going from the simplest element to the most complicated may represent, as a first approximation only, but quite logically, a curriculum of study for the training of mathematicians.

That is what we shall call, to serve as a reference for our later discussions, the teaching of mathematics for mathematics sake. Its organisation in the form of a continuous upward progression implies that each level reached will be a prerequisite for the level immediately following.

Such a curriculum does not exist in the pure state but it appears to be the foundation, the skeleton of most programs of general mathematical training in many countries, being a reflection of the European rationalist cultural tradition.

Adaptations of this consist essentially in heavier or lighter pruning, stretching to varying degrees the progression, or illustrating it to some extent by an appeal to real-life experience (either in order to introduce a notion or to demonstrate some application of it).

The first question raised then is whether such teaching is a suitable basis for mathematics for all.

On the level of objectives, the reply is obviously negative: The training of mathematicians can interest only a minute portion of students.

(ii) Selection by Means of Mathematics

In France, statistics show that of any 1,000 students entering secondary education, fewer than 100 will obtain seven years later a scientific baccalaureat (including section D) and a maximum of five will complete tertiary studies in mathematics or related disciplines (computer science in particular).

Referring again to statistics indicates that only about one successful candidate at the baccalaureat in six holds one of the types of baccalaureat (C or E) in which mathematics are preponderant. That fact, together with other indications concerning class counselling, brings out sufficiently clearly the importance of selection — a well enough known phenomenon anyway — by mathematics in the secondary school. This selectivity appears moreover to be relatively stronger than at university. This situation makes mathematics a dominant subject. French, which formerly shared the essential role in selection, is now relegated to a secondary position.

This selection is manifested most often by a process of orientation through failure for students at certain levels. But in fact, this sanction is usually only the deferred result of an ongoing selection process which takes effect cumulatively. From the primary school, or as early as the first years of secondary school, the classification between «Maths» and «non-maths» students becomes inexorably stratified.

In recent years, the idea that selection through mathematics is equivalent to a selection of intelligent students has made some progress, even if it is only very rarely expressed in such a clear way.

This function as the principal filter of the education system has considerably harmed the prime constructive function of mathematics as a means of training thought processes by the practice of logical reasoning. Just as a filter naturally catches waste, so mathematics produce academic failures inherent in the system, in other words, not due to intrinsically biological or psycho-effective causes but to the teaching process itself.

The type of evaluation used is not unconnected. It has the general fault of all standardised evaluation as is still too widely practised.

It supposes a definition of the child's normality that pediatricians and psychologists contests[1,2] : ranges of development, differences in maturity are just as normal and natural as differences in height and body weight. The same applies to the formation and development of abstract thought, which one must expect to be facilitated by the teaching process and not measured and sanctioned by it.

Aptitude for abstraction seems to be generally considered, with intelligence, as having an essentially innate character, whereas it is admitted by researchers that the share acquired in the social and family milieu and then at school is probably preponderant.

The demands of «Levels of intelligence» are also judged excessive for the teaching situation (first years of secondary school). They would necessitate[2] a clearly above average IQ.

On this subject we may note a very important lack of coordination between the quite reasonable programs and instruction of the Inspectorate General of Teaching and the contents of textbooks.

We shall see later some questions concerning the vocabulary used, but where the program considers only arithmetic or operations on whole numbers or rational numbers, it can be seen that in fact a veritable introduction to algebra is carried out.

(iii) Emotional Responses and Mathematics
All teaching is obviously subject to emotional responses: the student likes this, doesn't like that, prefers this, and so on. As far as mathematics is concerned, successful students acquire an assessment based on a harmonious relationship, but those who have difficulties feel strong emotions that can induce suffering and anguish.[4]

II. Problems of Language, Symbolic Writing

A mathematical apprenticeship requires the acquisition of a special language which is characterised by the interlocking of a conventional language (with nevertheless its own semantics and syntax) and a symbolic language.

If, beside their communicative aim, all languages serve as a medium of thought — according to Sapir: The feeling that one could think and indeed reason without language is an illusions — the language of mathematics, more than any other, is adapted to that very end. The sentence (containing words) and the formula with its symbols are vehicles of logical reasoning. In this area, symbolic writing is considerably more powerful than conventional writing: one can say that it is a motive force driving thought ahead more rapidly.

(i) The Power of Symbols
On the occasion of the 4th International Congress on the Teaching of Mathematics (ICME 4), Howsons[5] clearly showed the power of symbols, which one could have thought in the first analysis to be only tricks of abbreviation, whereas they do generate new meanings.

As essential elements of mathematics, they permit the discipline to develop without its being necessary to burden our thought processes with all the meanings with which they are charged. A language open to independent development, symbolic writing lends itself to operations the automatic nature of which, once it is acquired, saves conscious thought or at the very least permits considerable economies in the process of reflexion.

An apprenticeship in symbolic writing and the attendant operational procedures is therefore essential in the teaching of mathematics.

(ii) The Importance of Language Acquisition
The nature of symbolic writing being a capacity for self-development, if what has been learned in this area is already considerable, the student will have no major difficulty in acquiring the language necessary if he is to pass to the next stage. Thus his difficulties will reside rather in the structures of reasoning than in a knowledge of symbols. It may be considered that this is the case of students in the upper classes of secondary school.

On the other hand, at the beginning of this apprenticeship (notably when algebra is introduced), the change from the natural language to symbolic language, because it is a prerequisite, no doubt has a special place in the hierarchy of difficulties.

(Iii) The Difficult Changeover to Symbolism
The changeover from natural language to symbolic language, as well as the problems caused by too rapid or too early an introduction (poorly adapted to the development of the thought processes of the student and his maturity) carries with it some more technical difficulties, which in our view have not been satisfactorily solved.

Symbolic formulation is more than mere translation. The physicist is well aware of this, considering as he does today this operation, called (mathematical) modelling, as being of prime importance in the analysis of complex phenomena or systems. In the same way, the return from the formula to realist is an exercise that is not self-evident and a table of correspondences and a dictionary will not suffice.

Symbolism introduces first of all a complication. Afterward, naturally, when the obstacle is overcome, one profits as a result of a simplification of procedures (automatic responses in operations and their reproduction).

If one can solve a problem in ordinary language with a level of difficulty N1, to use for its solution a poor knowledge of symbolic language makes it more difficult (level N2). On this subject the tests of C. Laborde[7] seem significant. Confronted with solving concrete problems or describing mathematical objects, students do not use the codes they have learned. But once the symbolism is better known, the level of effort to attain the same goal is less. Level N3 is for example the level of effort required of the master mathematician. This summary demonstrates at the same time the advantage of learning mathematics and the difficulty there is, starting with the concrete description of a problem to formulate it in mathematical terms. It also shows that the teacher should give considerably more attention to lessening the difficulty of acquiring the mathematical metalanguage than accumulating purely mathematical knowledge.

(iv) The Necessity of Introducing Stages Useful for Conceptualisation
G. Vergnaud[8] has demonstrated that, when solving problems, students used Faction theorems» or implicit theorems, which were simply the products of their personal conceptualisation revealing the workings of individual thought processes. Several researchers have noted that such processes did not follow the shortest path of the mathematics taught nor the best method from the point of view of logical rigour.

Because of this, it is often considered by the teacher to be bad reasoning - to be done away with as quickly as possible in favour of classical mathematical reasoning — whereas it is rather logical reasoning in the process of developing.

The act of teaching, instead of ignoring or indeed rejecting the representation constructed by the pupil, his own personal mechanisms of thought, should consist, on the contrary, in revealing these, understanding them and using them.

As long as educational research does not provide practical ways of accomplishing this development, it is no doubt right to give to the acquisition of symbolic logic a more important share in the teaching process.

Inspiration may come from the evolution of symbolism in mathematics through the centuries.

Howson alludes to this[5] and the analogy of the evolution of the individual's knowledge according to Piaget's theory. Those noted in physics are arguments in the same direction; if one begins with the hypothesis that human logic can exist, it is likely that there are similarities.

But above all, so that the student finds his way naturally, we should propose to him varied representations of the same thing: "a supple and changing, suggestive and logical formalism " according to Lowenthal.[9] We come back to the recommendation of Howson and Brandsond: «no symbol or contraction should be introduced if the student is not ready fully and reasonably to appreciate the advantage it offers».

We consider that the use of natural language along with symbolic language can not only better guarantee the acquisition of the symbolic language[5,6] but above all serve as a better basis or guide for the logical reasoning associated with mathematical development.

(v) Avoidable Difficulties

As well as the intrinsic difficulties in the acquisition of the symbolic language of mathematics, there exist difficulties that one could avoid, growing out of the language used to mediate between natural language and symbolic language.

This is the language used by teachers or school text to give definitions, enunciate properties and theorems and to provide the necessary explanations for beginners.

The language used by teachers is obviously very diverse and varied, and there is no doubt that large numbers of them know how to adapt as is necessary. In France the General Inspectorate of Education encourages them to do so. It recommends in particular that they avoid the introduction of too many new words.

But if one considers school textbooks, one can ponder whether these instructions have been taken into consideration. The intellectual worth of the authors is not in question, and one must seek the reason in an insufficient realisation of the importance of the linguistic vehicle. We have used a textbook for the level known as «5e» where, exceptionally, a first chapter is devoted to helping in understanding the terms used in the body of the text. So as to draw a conclusion «a fortiori» we subjected this chapter to a test for the «classification of texts according to the difficulty of the approach required for understanding them» used in technical education to select documents for students according to their academic level.

This test has no pretentions to scientific perfection but the results achieved demonstrate its pertinence.

The result is edifying: With respect to the French used, this test should be given only to students three or four years older. The analysis of difficulties shows essentially:

1. that the vocabulary used includes too many words which are not part of the everyday language of the student;
2. that certain known words are used in different senses (paronyms);
3. that there is a supposition of certain references of experience (not only mathematical but also of a cultural nature);
4. that the structure of typical phrases aimed at mathematical precision causes ambiguities on the level of the French language.

As for the first two of these four observations, we carried out a summary evaluation of the vocabulary requirements of five of the most widely used textbooks. With respect to the first level of the basic French vocabulary (representing between 1,300 and 1,500 words) the comprehension of the French used as a vehicle for mathematics teaching (not including symbols) requires the knowledge of 100 to 150 new words or expressions.

In this body of material, the words that seem to be known but which are used with a different meaning created a doubly negative effect; they are not passive obstacles to comprehension, but introduce confusion.

Several researchers[10] have demonstrated this undesirable effect of the most common of these: if (and only if), then, and, or (exclusive), all ... These fundamental words should be introduced with the same care as symbols for they are not stepping stones to symbolic language, they are merely its image.

Elements supporting reasoning, they need to be perfectly assimilated so that the correct reasoning may be carried out. But they are not the only ones that cause the specialised language of mathematics to be in fact very different from natural language. Other less frequent uses, as well as syntax, increase the difficulty.

On this subject, one can raise questions (ii) concerning the origin of language difficulties in mathematics. Do they come simply from an insufficient mastery of the natural language? Such a deficiency obviously introduces a handicap.

But the quite widespread existence of students classified as «Literary» and «non-mathematical» shows clearly enough that it is not sufficient to know French better in order to understand mathematics.

Does not the difficulty of access to formal language also reside in the incapacity of natural language to translate it without weighing it down or even deforming it? This is obvious for the initiate, to whom the formula offers a richer meaning than the theorem that attempts to express it. The connection, indeed the interdependence between the mechanisms of formation of thought and of formal language, still insufficiently known (cf. different hypotheses of Piaget, Bruner, etc.) also lead to questions about an influence of one upon the other (and vice versa).

But the fact that the question is so open to discussion does not free the teacher from considering the more down to earth problems of vocabulary and syntax. One should

not write an introductory manual of algebra (or of other areas in mathematics that make considerable use of symbolic writing) without having the French corrected by a specialist in reading.
Thus one would create the most direct contact possible between natural expression and symbolic expression. And if it were realised—which is likely— that short-cut explanations, by means of Typical mathematical discoursed are not practical, perhaps one would attain, at the cost of an apparent waste of time, better comprehension.

III. Proposals for the Teaching of Mathematics for All Students

From this analysis of faults and difficulties result some paths that may lead to improvements in teaching.

(i) Restore the Role of Mathematics as a Tool
In our highly technological age, everybody no doubt needs some background in mathematics.
That is why the teaching of mathematics as a tool ought to be of interest to the majority on condition that, by taking certain precautions, it is made sufficiently accessible. On this condition it appears to us the only viable basis on which to found the structure of mathematics for all.
But what form can such instruction take? It could not be limited to a curriculum adapted to professional ends constructed on a basis comparable with that of basic teaching. For example, although the successive introduction to algebra and then differential and integral calculus can furnish a tool for the solution of problems of mechanical physics, if it does not gradually reveal its concrete basis and its applications we shall not consider it as instruction in mathematics as a tool.
The pedagogical procedure too often used consists of asking the student to acquire numerous prerequisites and to await the whole construction piece by piece of the cognitive edifice in order to perceive at last the end to which it can be put means the teacher is avoiding his responsibilities and it kills the student's motivation .
Teaching mathematics as a tool means giving permanent priority to the solving of problems and not to learning formal aspects of the discipline. Pedagogically there are two great advantages in this:

- we have seen that excessive formalism or too early an introduction of symbolism was an obstacle in the early stages;
- it is now allowed9 that the development of logical reasoning is carried out essentially on the basis of experience in problem solving.

The basic notion is to replace the upward progression in mathematics isolated by its formalism by a spiral progression dependent on other disciplines. This presupposes undertaking at each stage of initiation an adaptation of teaching methods inspired by research on language and conceptualisation[5,6,7,9,12].

- arouse interest in a problem 'set the right type of problem);
- bring the student to pose it in logical terms, to translate it into already familiar mathematical terms (modelling) and thus bring home the practical application of mathematics;
- give practice in the corresponding operations;
- show the polyvalence and indeed the universality of methods of logical reasoning, the utility of formalism;
- let the student measure from time to time the resultant enrichment of his capacities in the area to which the subject of the problem belongs;
- bring the student finally to a higher mathematical level.

(ii) The Place of Mathematical Modelling
One point that seems to us fundamental is the introduction of «modelling». Here again one can see the fruit of the physicist's experience, but such a procedure is in our view necessary more as a result of our earlier pedagogical considerations concerning the difficulty of acquiring symbolic language. «Modelling» or translating the concrete problem into pertinent mathematical terms does not come easily. The teacher must make a special study of the question:

- how does one, when faced with a more or less complex system, observe it, identify it, express its workings in formal relationships?
- how, when faced with a concrete problem, does one describe it, translate it into equations?
- how, thanks to the tools of mathematics, does one progress in one's understanding, one's solution of it?
- how finally, at the more sophisticated stages, does one iterate identification and modelling to the limits of one's own knowledge?

Thanks to such a process, the teacher will be able to facilitate the student's conceptualisation. As a result of special attention to the problem and the development of an open educative process, the teacher will be able to follow the student's «natural» principles of reasoning, reveal the formation of «theorems in action» mentioned above, and thus facilitate by an appropriate pedagogical method the development of these into true theorems.
Modelling thus leads into creativity and technological processes.

(iii) The Necessity of an Interdisciplinary Approach
An intensive use of modelling requires the mathematics teacher to have a good knowledge of the applications of mathematical tools in a variety of areas and makes it necessary for the development of his teaching to be kept in line with other disciplines using mathematics. This supposes not only a basis in team work, but also in a national curriculum and general interdisciplinary planning. A better solution would no doubt be the creation of true multidisciplinary subjects, an added advantage of which would be to link up again areas of knowledge that the division into disciplines has fragmented or simply overlooked.

(iv) Mathematics as a Tool and Mathematical Culture
There is no inherent conflict between mathematics as a tool and mathematical culture, one being able to lead to the other and vice versa. Restoring the teaching of mathematics as a tool will allow us to interest students and to offer them greater possibilities of success and self-development in the modern environment.

This is the path that seems the most certain to lead to mathematics for the majority, through a process of success and not of lowering standards.

(v) Teaching Through Goals with a Differentiated Progression
The working out of such a system of teaching would imply avoiding a drop in standards through an evaluative process based on objectives that clearly marked out the development of the curriculum, the chronology of which would be subject to modification and would permit the most gifted students to advance more quickly and those in difficulty to follow at a different pace.

It is accepted that between the beginning of secondary school and the baccalaureat the majority of students repeat a year once or twice, and this gives room in the curriculum and the means of attaining a differentiated progression.

The abrupt and penalising nature of repeating a year when one begins everything over again, even the things in which one has been successful, would be attenuated and greater consideration would be given to the timing of initiation, work and development.

It has always been accepted for a diploma like the baccalaureat that differences of level should be tolerated in various disciplines. Would it be fatal to experience failure in mathematics between preschool kindergarten and the A or B baccalaureat? Would it not be better to reach this stage by a well organised progression and natural orientation rather than in fits and starts with futile intermediate sanctions, since in the end students will reject or avoid mathematics if they cannot succeed.

Would it not be better to provide for success in slow stages, or related to more limited objectives, rather than to suffer failure so fully and so prematurely internalised that it leads numerous students and then adults to a veritable lack of mathematical culture?

Conclusion

Underlying the no doubt imperfect proposals presented above is a deeper question of objectives.

Will mathematics, rather than being a filter of the elite, recover their principal function of being the most wonderful of tools (albeit an immaterial one), of being the way of teaching logical reasoning?

By laying aside the attributes that make them forbidding (language, abstraction), by capitalising on their interest and power, mathematics could be accessible and interesting for the majority of students, who would all reach their appropriate level.

Finally, even in the event of only relative success, one could restore the supply of scientists and mathematicians that has dried up radically in recent years. Statistically one would no doubt also achieve a better quality elite.

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The Problem of Universal Mathematics Education in Developing Countries

Bienvenido F. Nebres, S. J.

In his paper «Mathematics for All— Ideas, Problems, Implications,» Peter Damerow underlines two central concerns. The first is that the canonical school curriculum for mathematics was designed for an *elite* and so there are serious adjustment problems when it is made universal. The second is that it was designed for a *European* elite and so the adjustment problems become even more serious when it is introduced into the mass educational system of a developing country.

«I think firstly we have to consider the fact that mathematical education in the traditional sense has its origin in a specific Western European cultural tradition, where the canonical curriculum of traditional school mathematics was created in the course of the l9th century. The transfer of this curriculum to developing countries in most cases has been closely linked with the institutionalization of schools by colonial administrations in these countries. It is well known that these schools generally were attended only by an elite, which adopted the Western European culture and often studied afterwards at European universities. Under these conditions it seemed natural simply to copy the curriculum of higher education. But it is quite another problem to build up a system of mass education in the countries of the Third World and to embed mathematical education into the specific cultural contexts of these countries. «[1]

In a paper entitled *Problems of Mathematical Education in and for Changing Societies: Problems in Southeast Asian Countries,* which was presented at the October, 1983 Regional Conference in Tokyo, Japan, I tried to classify mathematical education problems into two types, micro-problems and macro-problems.

«*We can classify problems of mathematical education into two types: the* first *we might* can micro-problems *or problems internal to mathematical education. These would relate to questions of curriculum, teacher-training, textbooks, use of calculators, problem-solving and the like. The* second *we might* can macro-problems. *These are problems affecting mathematics education because of pressures from other sectors of society: economy, politics, culture, language, etc. One of the features of a* developed *society is a reasonable differentiation of sectors and functions of society. While given sectors are, of course, interdependent and affect one another, they also have some reasonable autonomy. School budgets may increase or decrease, but they have some stability and so it is possible to plan. Teachers get a sufficient {though not high} salary so they can concentrate on their teaching chores. But in contrast, structures in developing societies are not sufficiently developed to provide (for example) education and culture with sufficient freedom from the pressures of politics and economics Teachers may be called upon to perform many civic duties - to the detriment of their classroom work. Their salaries may nor be sufficient for them to be able to concentrate on their work. Budgets may be unstable and information and opinion tightly controlled ,* «[2]

In that paper I discussed the problem of universal mathematical education for developing countries, mainly in terms of economic constraints.

«The problem I would like to concentrate on here is that of the great number of students who are in school only for four to six years. One must, therefore, give them functional numeracy within severe constraints. The time constraint is obvious. There are also problems of scarcity of textbooks, not-so-well-trained teachers, language. We might focus the question on only one aspect of the problem: curriculum. In the Philippines, at least, the curriculum is the same whether a student goes on for ten years through high school (or even beyond to university) or whether the student stops after four to six years. I propose the following questions. From a study of the curriculum and from experience, at what point is functional numeracy realistically achieved? After four years? After six years? After eight years? If one were to look at the curriculum from the point of view of best helping a student who will stay only for four to six years, would one redesign the curriculum?»[3]

However, on further reflection it seems clear that the deeper problem is, as is noted by Peter Damerow, cultural. «So I think the relations between mathematics and culture is the first and maybe the most general question which arises when mathematics for all is taken as a program. «[4] For developing countries, the problem of mathematics education and culture may best be understood by reflecting on the history of the school system in these countries.

«All of the countries of Southeast Asia, with the exception of Thailand, went through a prolonged colonial period. During the colonial period, the school system was patterned exactly after that of the colonising country. The norms of fit between school and society were quite precise: the school system was to come as close as possible to that of the mother country. It should produce graduates that would fit into the civil service and who would do well in universities in the mother country. With independence the above norms of fit between school and society were seen with mixed feelings. Leaders became conscious that a school system developed according to such norms would, among other things, simply contribute to the brain drain. They also became conscious that the school system had to respond to different cultures and classes in the country: a westernized elite, a growing lower middle class, urban workers, a traditional rural sector. The aspirations for progress and equality led to new questions about the role of the school system in society:

- Can the school system provide functional literary and functional numeracy to the great number who attend school only for four to six years?
- Can the school system provide the scientific and mathematical skills for different levels in the agricultural, commercial, and industrial work force?
- Can the school system train sufficiently well the small but important number needed for leadership in the scientific and economic sectors?

These are, of course, very difficult tasks. The specific problem faced by the school system in many developing societies is that the society at large expects it to fulfill the society's dream of progress and equality. These place unrealistic pressures on the school system.»[5]

1 Damerow, P. (1984): Mathematics for All - Ideas, Problem, Implications. In: Zentralblatt für Didaktik der Mathematik. No. 3, pp. 82-83.
2 Nebres, B. (1983): Problems of Mathematical Education in and for Changing Societies - Problems in Southeast Asian Countries. In: Proceedings of the ICMI-JSME Regional Conference on Mathematical Education, Tokyo, p. 10.
3 Ibid., p. 16.
4 Damerow, P., loc. cit.
5 Nebres, B., op. cit., p. 12.

I. The Lack of Fit Between School Mathematics and the Socio-Cultural Context of Developing Countries

There have been some very interesting examples in the papers presented for the theme group «Mathematics for All» at Adelaide regarding the lack of fit between school mathematics and socio-cultural context.

In the paper «Having a Feel for Calculations,» several examples are given of young street vendors using non-school algorithms to do fast and accurate calculations.

«Customer: How much is one coconut?
Vendor (12 years old, 3rd grade): 35.
Customer: I'd like 10. How much is that?
Vendor: Three will be 105, with 3 more, that will be 210,
I need 4 more ... that is 315 ... I think it is 35O.»[6]

Yet these same young people did very poorly in the same calculations in the school setting. The paper concludes:

«There appears to be a gulf between the rich intuitive understanding which these vendors display and the understanding which educators, with good reason, would like to impart or develop. While one could argue that the youngsters are out of touch with the formal systems of notation and numerical operations, it could be argued that the educational system is out of touch with its clientele.»[7]

In another paper on «Mathematics Among Carpentry Apprentices,» Analucia D. Schliemann compares the performance and computational methods of professional carpenters with apprentices. What was most striking was the fact that the apprentices insisted on following «school» procedures even when a little reflection would have shown them that these were, in practice, absurd. fit seems that the task was approached by the apprentices as a school assignment and they did not try to judge the suitability of the answers.»[8]

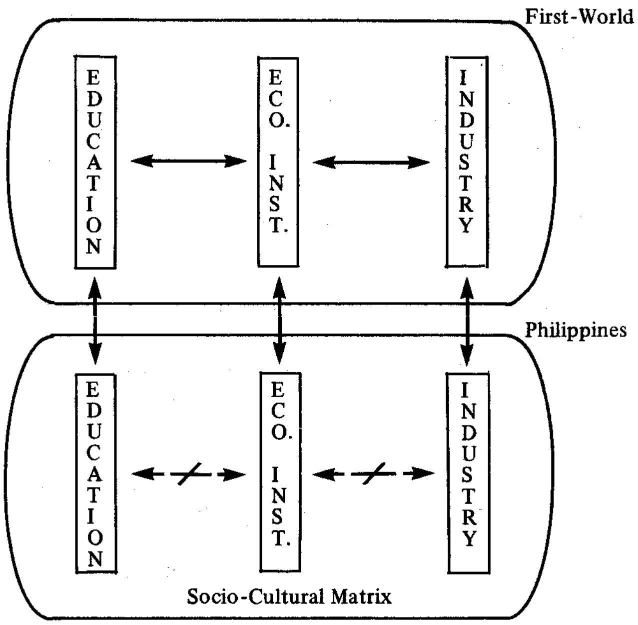
In an earlier discussion on universal primary education, I had noted our failure to respond to an immediate need of farming communities throughout the Philippines.[9] The introduction of high-yield varieties of rice brought in, of course, greater productivity. However, it also demanded higher inputs in terms of fertilisers, pesticides, labor or machinery for weeding. Farmers had to take out loans to avail of this new technological input. The farmers were lost in the new economics of the system. As many of them put it, «I know I am getting bigger harvests. But I also know I am sinking deeper into debt.» Our school system in the rural areas continued happily teaching sets and commutativity, oblivious of the need for simple bookkeeping.

The plenary address of Ubiratan d'Ambrosio on *Ethno-Mathematics* places this discussion in an even more fundamental setting. Can we develop mathematics in countries with different cultural traditions, which may be quite different from the mathematics developed in Greece and Western Europe? Would such a mathematics serve the needs of other cultures better?

There are other things we could do to understand better the gap between school mathematics and our socio-cultural context. I have, for example, studied the textbooks generally in use in the Philippines. They are either direct copies or relatively mild modifications of textbooks in the Western world. There is little awareness that there is a different context outside. I have also analyzed test items given in an assessment study of sixth graders throughout the Philippines. There were 40 items, 10 on computational skills, 12 on concepts such as place notation, 10 on routine-type applications, 8 on analysis of data. They are the usual types of exercises we put in textbooks to develop manipulative skills. The problem is that most of the concepts or skills developed would have no relevance for the young person dropping out of school after six years.

II. The Historical Development of Institutions in Developing Countries

The above analysis highlights the serious gap between school mathematics and the socio-cultural context of developing countries. I would like to locate the primary cause of these problems in the history of our social and cultural institutions. The first I would call *vertical,* that is, the relationship between *similar* institutions, like schools, in *different societies.* The second I would call *horizontal,* that is the relationship between *different* institutions in the *same country.*



6 Carraher, D., Carraher, T., and Schliemann, A.: Having a Feel for Calculations. ICME 5 Mathematics for All Collection, p. 2.

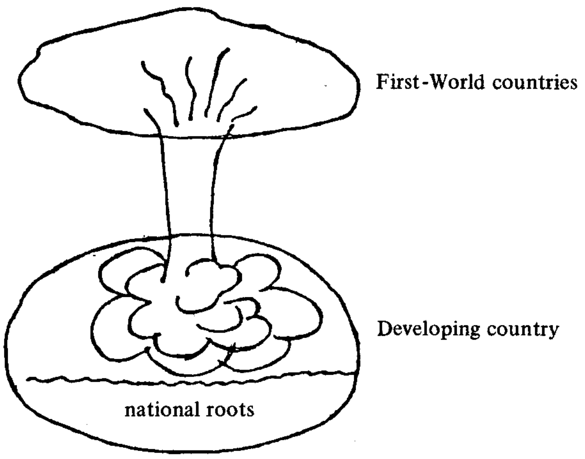
7 Ibid., p. 6.

8 Schliemann, A.: Mathematics Among Carpentry Apprentices: Implications for School Teaching. ICME 5 9 Mathematics for All Papers, p. 7.

9 Barcellos, A. (1981): Universal Primary Education. Teaching Teachers, Teaching Students. Steen & Albers eds., Birkhäuser, p. 123.

I would also add a third relationship of *rootedness,* that is, the insertion of these institutions in the socio-economic-cultural matrix that underlies the given society.

To understand the situation, we should note that the history of social, industrial, educational institutions in most developing countries has been guided mainly by *vertical* relationships. For example, to understand the school system in the Philippines, it is less necessary to understand the local social and cultural situation as it is to understand the American school system and to note the adaptations that have been made. If one were looking at Malaysia, one would turn to the British school system. The same can be said regarding the system of hospitals and health care, financial institutions, etc.



If we were to picture the development of institutions like that of a tree, where the institutions represent the leaves, branches, fruits (the visible developments in society), then we would picture our institutions like upside-down trees. They are rooted not so much in the socio-cultural matrix of the country, as in the socio-cultural matrix of model countries abroad. This whole pattern of development of institutions according to *vertical* relationships has produced what is usually called the modernized sector, which includes the best of the educational system. This modemized sector is not a product of the socio-cultural matrix of the country, but is very much a foreign transplant. Much of the air it breathes is imported air, whether this be imported curricula, imported talent, or imported management techniques.

What are the consequences of this development guided mainly by vertical relationships?

(1) The development of our institutions is directed by their sources abroad, not by complementary institutions or needs in our society. Thus a leading educator is invited to an international conference and is introduced to the new mathematics or to computer assisted learning or distance educational techniques and he comes back and wishes immediately to implement what he has learned. Because it is the latest and the best, whether or not it has serious relevance to the country. The norms for judging the value of our schools are often in terms of how well our students do in graduate school in England or the United States, rather than in how they fit and contribute to the larger society.

(2) The analysis in the earlier part of the paper shows that *horizontal* relationships and *insertion* into the local culture are weak. The mathematics classroom is totally unaware of the «street» mathematics of the young pupils inside. There is no linkage between the needs of a rural community for better bookkeeping and the new mathematics being taught to its children.

These results define for me a crucial task for the future: how to develop better horizontal fit and better rootedness in the socio-cultural context.

III. Tasks that We Might Attempt to Improve the Situation of Universal Mathematical Education in Developing Countries

I would like to propose *two tasks.* One is in the area of bringing about a cultural shift in our countries. The second is a more specific task of working towards a better integration between universal mathematical education and the outside world to which our students will go.

(1) Bringing about a cultural shift in our countries

I would propose that mathematics educators, together with other educators and other leaders of society, take up this need of having the social and cultural institutions of the country be better integrated with one another and be better inserted into the culture. This is not to deny the importance of linkage with other institutions in the Western world or in the other developing countries. It is simply to accentuate the need to have these imported developments be integrated into the social and cultural milieu of the country. It is important for us to accentuate the high cost to development of this lack of integration. Whether the cost be in terms of brain-drain or in terns of graduates who cannot find jobs in our society. Or a young population without the skills for a productive life.

(2) How to proceed concretely to bring about a better integration between the universal mathematics curriculum and the world into which our students will be going

In the proceedings of the Fourth Intentional Congress on Mathematical Education, Shirley Frye has a suggested mathematics curriculum for students who leave school at an early age.

«The particular goals of a minimal mathematics education include having:
1. a sense of number;
2. the ability to quantify and estimate;
3. skills in measuring
4. usable knowledge of the basic facts
5. the ability to select the appropriate operation to find a solution;
6. the ability to use a calculator to perform operations
7 . a 'money-wise' sense.

The last skill relating to being 'money-wise' is most important since an individual should have the ability to decide whether wages are being paid correctly and if purchasing transactions are fair. «[10]

The proposal is that mathematics educators in our country attempt the following tasks:

(1) Study the actual articulation in the curriculum of the seven goals stated by Shirley Frye, that is, how are they translated into mathematics concepts, into mathematical skills, into problems in the textbooks.

(2) Look for how these goals appear in the social environment of the student. That is, what tasks or events bring about a sense of number or require the ability to quantify and estimate and so forth (cf. Carraher and Schliemann papers).

(3) We should then compare the two, that is, the curriculum inside the school and the appearance of these goals and concepts in the outside world and see how we can bring about a better integration between the two.

(4) Share results with one another and detemmine to show progress in ICME 6.

10 Frye, S. (1983): Suggested Mathematics Curricula for Students Who Leave School at Early Ages. In Praceedings of the Fouth International Congress on Mathematical Education. Zweng et al., eds., Birkhäuser, p.32.

Conclusions Drawn from the Experiences of the New Mathematics Movement*

Peter Damerow and Ian Westbury

Introduction

«Mathematics for All» is the title of our theme group. The group's implicit goal is the consideration of an issue which is fundamental to the idea of general education - and to be «successful» the theme has to be engaged in a way that goes beyond the limitations which appear to be endemic to contemporary curriculum studies: preaching the necessity of a new program and then arguing in hindsight that widespread adoption was, from the beginning, an unrealistic expectation. Needless to say, no such goal can ever be achieved in one discussion, but the issue which the group canvasses does not go away because it is a difficult one to address.

It is easy to make pointed comments on the emptiness of the kind of conclusion from curriculum research we parodied above. But at the same time, there is a sense in which such a conclusion is an inevitable result of a web of problems that all educational reformers, and particularly subject-based reformers, have faced as they think about the scope of their work and their agendas. *There is a need to find ways of changing the contents and conditions of general education as part of a larger concern for changing the fit between the work of the schools and the rapidly changing scientific and social demand for qualifications,* but how this is to be done is clearly totally elusive.

Within mathematics education the history of the so-called Knew» mathematics is one instance of this larger issue. There was a world-wide movement to introduce a Knew» mathematics—but we know that the effect of these efforts was negligible: Little has changed in classrooms and the change that has occurred bears little relationship to the goals of the original reform movement. This fact defines the parameters of our problem. Changing the fundamentals of general education in a goal-oriented, systematic, time-limited way poses innumerable unsolved issues. It may be true that the needs of rapidly-changing societies do not allow us to base the contents and the practices of general education on tradition and the inner experience of the school. It may be true that it is the task of our times to move the school system from being a relatively autonomous, developing social system into a guided institution which is continually adapted to changing needs by planning decisions and administrative action. But the fate of the new mathematics movement shows that so far we only know what the task is. We have not created the means needed to address it.

Mathematics for AID - As a Program of Reform

The reason for the presence at ICME 5 of a group discussing mathematics for all and the absence of a group addressing how new mathematics might be introduced into the school is intimately related to the experience of the new mathematics movement. At the height of that movement it was commonly assumed that the new mathematics *was* a mathematics for all. It was, for instance, the first addition to traditional arithmetic and so the first alternative to the Folds curriculum of the universal primary school for about 200 years. But as Damerow et al. have shown, the claims that were sometimes made for the real-world relevance of the new mathematics, the claims that would have been needed to establish a belief that the new mathematics had a real-world relevance, were often even more difficult to sustain than those associated with traditional mathematics.[1] More important, the «new mathematics» showed decisively how problematic major change in a subject can be: While there were some admirable experiments which showed what might have been done in elementary schools under the name of new mathematics, «experimental» outcomes were not generalized to school systems as wholes. Perhaps the best that can be said about the widespread introduction of the new mathematics was that its teaching did not inhibit the traditional teaching of arithmetic too much.

And what was the result of the implicit, though often tacit assumption of the period in which new mathematics was the vogue that the yield of a a "subject" reform could be secured equally in every culture independent of the degree to which formal education was institutionalised? There can be no doubt that Dienes' efforts to introduce a modern mathematics project in Papua-New Guinea in the mid-1960s was as successful as he claimed to be. But 20 or so years later, after intensive attempts to adapt Dienes' curriculum to local conditions, Souviney comments, in his discussion of mathematics education in Papua New Guinea, that it is not enough for the educational establishment «simply to institute a selection procedure which identifies and promotes children who exhibit 'high' educational potential while failing to address adequately the needs of the vast majority.» «Increased attention must be paid to the needs of [the group of children who will return to their villages after completing community school] who presently constitute 70% of the community school graduates.»[2] By implication, something much larger than the new

* An enlarged version of this paper is published in the Jour nal of Curriculum Studies, 17, 1985, pp. 175 -184.

1 Damerow, P. et al. (1974): Elementarmathematik: Lernen fur die Praxis? Ein exemplarischer Versuch zur Bestimmung fachuberschreitender Curriculumziele. Stuttgart: Klett.

2 Souviney, R. J. (1983): Mathematics Achievement Language and Cognitive Development: Classroom Practices in Papua-New Guinea. In: Educational Studies in Mathematics 14.

mathematics seems to have surfaced in the context which rendered the Dienes' «modern» program moot.

This instance is from the developing world but parallel instances can be found in the industrialized world. It is cultural and contextual factors which, as they interact with mathematics itself (or any subject area), pose the most serious problem which slogans like «new mathematics and «mathematics (or science) for all» must face. Do we keep, for example, the highly selective frameworks and methods of traditional mathematics education but give up the privileged position of the subject as part of the core of general education? Or do we seek to keep mathematics at the core of the curriculum but find a way of teaching the subject to all students?

Two Alternatives

What might these alternatives mean — for ideology and for reality? The first possibility would make « mathematics» a subject of early specialization, with the present role of the subject being taken over by physics, technical education, economics, etc. In this way most students would experience mathematics as a useful tool and concentrate on creative mastery of and application of the problem-solving techniques which result from mathematical thinking. The core of mathematics, its ideas, conceptual structures, methods of proof and the like, would only be taught to those who specialize in some way or other in the subject.

This suggestion comes close to the actual situation of mathematics education in many nations. In the Federal Republic of Germany, for example, only 11% entering university students enroll in the subject areas of mathematics and natural sciences; 21% enter programs in engineering and the remaining 68% range over all other fields. For most students, therefore, mathematics is but a potential tool, and all we are saying is that mathematics programs in school might reflect this situation. But mathematics in Germany is not taught in this way and most mathematics teachers would probably deny the possibility and would instead emphasize the specialist, pure mathematical aspects of their work. It is worth, however, mentioning that the alternative possibility we sketched above once played a substantial role in curricular thinking. When the influential «Verein zur Förderung des mathematisch-naturwissenschaftlichen Unterrichts» (Association for the Support of Mathematics and Science Education) was mooted in 1890, the majority of its potential members argued *against* a continuing role for pure mathematics as a core subject in the high school curriculum. At the founding of the association in 1891 a motion was passed against the teaching of pure mathematics. It was only as a result of the later influence of the Gottingen Professor of Mathematics, Felix Klein, that this policy was changed. But Klein promised that, in the near future, there would be reintegration of mathematics with its applications in other sciences and practices and therefore the continuation of the traditional kind and place of mathematics education was justified as an interim measure. The possibility of such a new mathematics became the goal of the association - and this turn-of-the-century anew mathematic» was profoundly influential both in Germany and internationally. Klein, of course, did all that he could to promote the development of such a mathematics with its implied integration with the domains of practice but he failed and given this it can be argued that the case for the abandonment of a «pure» mathematics for all is still as relevant today as it was in Gemmany in the 1890s.[3]

The second alternative we mentioned above is keeping mathematics as a fundamental part of the school curriculum but finding a way of teaching it effectively to the majority. What problems must be faced as we contemplate this?

We have first to consider the fact that mathematical education in the traditional sense had its origins in a specific cultural tradition. The canonical curriculum of Traditional mathematics» was created in the l9th century as a study for an elite and this pattern persists. In Germany, for example, «advanced» school mathematics (i.e. analytical geometry and calculus) are only offered in Gymnasium which in 1981 enrolled only 10.9% of the 18-year-old age cohort. And as enrolments in Gymnasium have increased it has seemed necessary to relax the once-fixed expectation that all students in the Gymnasium would complete a full program in school mathematics in order to maintain «standards.» In 1977—78 only about 70% of German students in grade 12 (the second last year in the Gymnasium) were taking the traditional sequence in mathematics, i.e. about 10% of the relevant age cohort.[4] While the particular curricular patterns of different societies vary, mathematics is constructed in most places in ways that lead to few of the students who begin mathematics in the early years of the secondary school continuing to take it in their last secondary years. The separation of students into groups who are tagged as «able-mathematically» and, «less abler is endemic. The heart of mathematics teaching is, moreover, widely seen as being centered on this curriculum for the able - although all students *begin* the study of mathematics. There are some important differences between countries in their retention rates but in the main we see the patterns which were created in the l9th century still holding; advanced mathematics is a study for a few.

The transfer of the European mathematics curriculum to developing countries was, of course, closely associated with the creation of schools for elites by colonial administrations. Under these circumstances it seemed natural to simply copy

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3 Lorey, W. (1938): Der Deutsche Verein zur Förderung mathematischen und naturwissenschaftlichen Unterrichts, E. V. 1891-1938. Ein Rückblick zugleich auch auf die mathematische und natur vissenschaftliche Erziehung und Bildung in den letzten fünfzig Jahren. Frankfurt a. M.: Salle.

4 Steiner, H.-G. (1983): Mathematical and Experimental Sciences in the FRG - Upper Secondary Schools. Arbeiten aus dem Institut fur Didaktik der Mathematik. Universität Bielefeld, Occasional Paper 40.

European patterns — but, as Souviney makes clear, it is quite another problem to build a system of mass education in the Third World and embed mathematics education in the specific cultural contexts of that world.[5] How is this to be done? Is a mathematics curriculum desirable if it causes students in these countries to develop the antipathies against mathematics which are commonly found in Europe—but in social contexts which lack the culturally-based consensus found in Europe that abstract mathematical activity is as good as such and must therefore be supported even if it seems on its surface to be useless. Even if this argument is inappropriate, it does raise the question of the relation between mathematics and culture which may be the first problem which arises when the idea of mathematics for all is raised as a platform for a program of action.

We must consider, second, the problem of conceiving, even for industrialized societies, a mathematics which is appropriate for those who will not have contact with pure mathematics after their school days. Most current attempts to face the problem of a basic, «minimal-competency» curriculum reduce the traditional curriculum by pushing out every mathematical idea and every possible difficulty to make it feasible to teach the remaining skeleton to the majority. But there is only a limited basis for an appeal to «utility» as an argument or a rationale for curriculum building to support this approach. *Students who* win *not have to deal with an explicit pure mathematics in their adult lives but will face instead only the exploitation of the developed pro - ducts of mathematical thinking {e. g program packages) will only be enabled by mathematics instruction in school if they can translate the mathematical knowledge they have acquired into the terms of real-life situations which are only implicitly structured mathematically.* Very little explicit mathematics is required in such situations and it is possible to survive without any substantial mathematical attainments whatsoever.[6]

But is this kind of argument a way of making the case for the first alternative we considered earlier? And is that alternative the only one we might be left with? It may be, but if this is true it would seem to deny the significance of the topic we are concerned with. Thus we might observe that to draw this kind of conclusion is to look backwards in order to determine educational aims for a future. The facts we have cited suggest that a program of mathematics for all implies the need for a *higher* level of attainment that has been typically produced under the conditions of traditional school mathematics—and this is especially true for mathematics education at the level of general education. To put this another ways we might claim that *mathematics for all has to be considered as a program to overcome the subordination of elementary mathematics to higher mathematics, to overcome its preliminary character, and to overcome its irrelevance to real-life situations.*

The Mathematics Classroom

A number of recent research studies have suggested that it is very likely that the structure of classroom interaction itself creates ability differences among students which grow during the school years. What might cause these growing differences in mathematical aptitude? The simplest explanation rests on the assumption that these differences are due to predispositions for mathematical thinking - with the implication that nothing can really be done to change the situation. But this explanation is too simple to be the whole truth.

The understanding of elementary mathematics in the first classes of primary school is, we know, based on preconditions like the acquisition of notions like conservation of quantity which are, in their turn, embedded in exploratory activity outside the school. As long as the genesis of general mathematical abilities is as little understood as it is, the possibility that extra-school experience with mathematical or premathematical ideas influences school learning cannot be excluded. Furthermore, we know from classroom interaction studies that the differences between intended mathematical understandings and the understanding which is embedded in normal classroom work is vast. We cannot exclude the possibility (aggressively suggested by Lundgren) that classroom interaction itself in fact produces growing differences in mathematical aptitude and achievement by a system of positive feedback mechanisms which increase high achievement and decrease further low achievement.[7]

Such classroom level phenomena also interact in profound ways with curricular factors. The English Cockcroft Committee on teaching of mathematics pointed to the significance of the notion of *curricular pace* as a critical variable affecting school achievement. If a pace necessary to cover an overall curriculum (i. e. to reach the levels of understanding necessary for, say, English sixth-form work) is to be sustained, a given rate of coverage is required of teachers. The Cockoroft Report claims that in England there has been little change in this implicit rate since the pre-war years despite the fact that the cohorts of children ostensibly learning mathematics are now drawn from the second and third quartiles of the general ability distributions (as a result of increased access to secondary schooling). The result, the Cockcroft Committee has suggested, is an overall rate of coverage and pace of instruction which is far too fast for many if not most pupils. For such pupils math-

5 Souviney, R. J. (1983): See Footnote 2.

6 See, for example, Bailey, D. (1981): Mathematics in Employment (16-18) (University of Bath); Sewell, B.: Use of Mathematics by Adults in Daily Life: Enquiry Officer's Report (Advisory Council for Adult and Continuing Education, London); ADVISORY COUNCIL FOR ADULT AND CONTINUING EDUCATION (1982): Adult's Mathematical Ability and Performance. London: ACACE.

7 Lundgren, U. P. (1977): Model Analysis of Pedagogical Processes. Stockholm Institute of Education, Studies in Curriculum Theory and Cultural Reproduction 2. Lund: CWK Gleerup.

matics as a subject is abstract, mechanical, and procedure-based, and success is hard to come by.[8]

Implications

Our discussions to this point make it clear that to talk of mathematics for all entails an intention to change general attitudes towards mathematics as a subject, to slow the pace of teaching, to eliminate divisions between those who are friends of mathematics and those who are not, to diminish variance in the achievement outcomes of mathematics teaching. This, in its turn, involves us in an analysis of the forces found in social contexts, curricula, and teaching inasmuch as it is these forces together which create a set of frames which *create* situations in which mathematics becomes one of the subjects in the secondary school in which *selection* of students into aptitude and ability groups is an omnipresent reality from almost the earliest days of secondary schooling.[9]

As we ponder what such notions might mean we have to address three very different levels of analysis of the mathematics curriculum.

1. The distribution of knowledge. With the implication that we reject assumptions that mathematical knowledge is the prerogative of some cultural communities and not others and instead see mathematics as something potentially appropriate to all people. At this level the idea of mathematics for all involves issues of cultural exchange and intercultural understanding - within and between social groups and geopolitical communities.

2. The school system and its integration into the society. The idea of mathematics for all poses an issue of general education rather than elite education. At this level the idea of mathematics for all involves us in a rethinking of the traditional concerns of mathematics education—away from the «needs» of elites and towards the needs of both elites and average students; our sense of crowning achievements would come not from the achievements of the few but from the achievements of the many. Our index of accomplishment would be the overall *yield* of the school system (i. e. the percentage of a cohort mastering given bodies of content and skill) rather than content and skill achievement of the most able.[10]

3. Classroom interaction. Mathematics for all is a problem of opportunities to learn and their relationship to the dynamics of the learning process. This level of concern must include an analysis of the assumptions, patterns, and practices of within-school division of students into ability groups, sets and streams — for setting/streaming is ubiquitous in mathematics education from the early secondary years.

It is quite clear, of course, that these levels are very closely linked together and that they serve to do no more than *define* the dimensions of the complex but coherent problem labelled by the slogan «mathematics for all.»

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8 Cockcroft Committee (1982): Mathematics Counts. Report of the Committee on Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr. W. H. Cockcroft. London: HMSO.

9 For an English study see Ball, S. J. (1981): Beachside Comprehensive: A Case Study of Secondary Schooling. Cambridge: Cambridge University Press.

10 For a conceptualization of «yield» see Postlethwaite T. N. (1967): School Organization and Student Achievement. Stockholm Studies in Educational Psychology 15. Stockholm: Almquist & Wiksell.

Implications of Some Results of the Second International Mathematics Study

Howard Russell

Peter Damerow has introduced some important suggestions which must be considered carefully if there is to be any progress made towards the goal of «mathematics for all» in the classrooms of our various countries.[1] His suggestions regarding pace and coverage are grounded in discussions and presentations which have been summarized in Mathematics Counts[2] and they have emerged as debatable issues in the period of the mid-eighties. The present paper is offered as a contribution to the debate and the data presented constitute an important part of a recently established data set.

The Second International Mathematics Study (SIMS3) data constitute this data set and these data show substantial variations among countries in the extent to which mathematics education is provided for all students. The data which reveal these variations are coverage data and retention data and the product of these two, i. e., coverage x retention, are presented as «yield.» While it is true that there is not much new in the concepts involved, the data are sometimes revealing when presented in the new form used in SIMS. These data may prove to be helpful as we proceed to consider a shift in mathematics education from content for the elite, to a more marketable, mathematics-for-all. The SIMS data may be helpful because they foreshadow relationships among key variables which need to be manipulated if the suggested shift in mathematics education is to take place in the mathematics classrooms of the world, as opposed to taking place only in the minds and the writings of educational leaders.

In this paper I propose to consider the SIMS pop A data first, i. e., from the 13-year-olds, and to use these data to suggest that already we have mathematics-for-all at the elementary level in many countries. There are variations in what mathematics is, at this level, but whatever it is, all youngsters get it. The SIMS pop B data, i.e., from 18-year-olds more or less, show signs that mathematics for an elite is still the prevailing plan in most countries. It is at this age level that some countries have tried to move towards mathematics-for-all and comparisons of these countries with the others which have not moved so far gives us good information with which to plan future strategies for implementing a mathematics-for-all curriculum throughout the secondary school years. But first let us look at pop A where, although it is true that virtually all youngsters are retained in school, and in mathematics, it is nevertheless clearly evident that different amounts of mathematics and different parts of mathematics are taught.

The pop A coverage data are shown in Table 1. These are topic by topic coverage data which are presented in country rows (see Figure 1). Thus, the coverage index for arithmetic for Country D is .74 and the standard deviation is .12. This means that the average teacher with a pop A class in Country D claims his/her students have been taught the material involved in 74% of the items under consideration. The standard deviation is the measure of variation in coverage C among teachers in Country D on that particular set of arithmetic items.

Since it is true that virtually all youngsters in a nation eventually make it through the pop A level, the variation in coverage is the main source of vacation in the mathematics which is offered to pop A students. The data in Table 1 show substantial variation from country to country, and as well the standard deviations show sizeable variations among classes within countries. It is true then that there are many youngsters who miss out on instruction in a substantial amount of the content defined by the SIMS Pool, even in such popular topics as arithmetic. Since this is a phenomenon which affects all countries, it may be one of the features of the present mathematics curricula that is difficult to change. This would be especially interesting if it can be shown that the youngsters who miss out on coverage are the ones who cluster in classes which spend time on unlearned content of earlier grades and/or require more than the usual amount of time to cover most topics. If this is true then it would appear that the teachers of slow classes have adjusted the pace to the needs of their students.

Although retention is uniformly high through the pop A years, it is nevertheless true that there is considerable variation in the amount of time taken by students to get through to the pop A level. This suggests variation in pace. Table 2 shows the mean age in months for both the pop B and the pop A students. The first message which emerges from this table is that some countries appear to take much less time to get through to the end of the pop A year than others. Country A requires only 162 months. Country K requires 166 months, and the other countries spread themselves over a range of many months. What interpretation such data have for deliberations about mathematics-for-all may be clearer when the associated p-values for student performance are presented at some future point in time. In their absence it appears that mathematics-for-all can be pursued as effectively by following the lead of countries with a low mean age as those with high mean age.

1 Damerow, P. (1984): Mathematics for All - Ideas, Problems, Implications. Paper presented to the ICMI Symposium at the International Congress of Mathematics. Warsaw, August, 1983. Zentralblatt fur Didaktik der Mathematik, 16, pp. 81-85.

2 Mathematics Counts (1982): Report of the Committee on Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr. W. H. Cockcroft. London.

3 Travers, K. et al.: The Second International Mathematics Study, Vol. 1, (for publication 1985) Pergamon Press.

The above argument seems to be in conflict with the hypothesis proposed by Cockoroft[4] that the pace of mathematics education must be slowed down if we are to keep enough students in the mathematics courses to accurately label it mathematics-for-all. Table 2 showed us there is wide variation in pace because there is wide variation in age among counties. If pace were related to retention at pop A or retention later then the age variation should be somehow related to retention. We have made the supposition that pace, for our purposes, is really coverage divided by time (age in months) and we have found that pace at pop A is unrelated to retention or any other variable of central interest. What seems to have happened then is that most contries have maintained a «promotion-by-performance» standard policy and this, in turn, has lead to retardation or failing of significant numbers of students. The failing of students, or the forced repetition of grades by students thus shows up a slowing of pace, but this type of slowing the pace seems not to have provided any positive outcomes. Another way of stating it is that no violence is done to the concept of mathematics-for-all when age promotion rather than the slower paced, promotion-byperformance standards is adopted at the elementary grade levels. Indeed, it can be postulated that violence may be done to the concept of mathematics-for-all if annual promotion-by-performance standards has the effect of retarding student progress to the point that many students give up schooling and mathematics the minute that they attain the legal leaving age. Clearly, more data and more study are required before such generalisations can be widely accepted.

The story is different at the pop B level. Table 3 shows the coverage data means and standard deviations. Again, there is wide variation both among contries and within. But these coverage data cannot speak to the issue of mathematics-for-all until they are augmented by the retention data. Table 4 shows coverage, retention and yield data for each country which possessed the necessary data. Now the variation among countries is even more evident, and the possibility of finding relationships seems promising.

In order to get one central relationship quickly we can look back at Table 2 which shows the mean age for the students in pop B. Along with these mean ages I have introduced the difference in ages between pop A and pop B. Now it seems evident that the countries which are attaining the highest yields are those which provided their students with the longest time interval between pop A and pop B. namely Countries E, O. A and K. What emerges then is a tentative conclusion which supports the Cockcroft hypothesis and as well reinforces the Damerow position elaborated in his paper on mathematics-for-all. It must be conceded that Countries C and F possess high yield, but a more thorough analysis of these countries in the Volume I of SIMS indicates that Country C has the lowest level of attainment by far and hence what is taught is no indication at all of what is learned. Country F also pays a price of low student attainment, and for purposes of the present argument that country as well as Country C is discounted. The SIMS data not only offer assistance in our debate by providing empirical support for the Cockcroft and the Damerow positions but also they help us to identify at least tentatively some of the boundaries beyond which this central pace hypothesis may be falsified.

Perhaps the first boundary worthy of attention is the one which separates pop A from pop B. On the surface it appears that the «slower pace produced higher yield» hypothesis breaks down between pop A and pop B. An alternative explanation of the data may be that the type of slowing down which is produced by annual performance promotions, i.e., failing the lowest students, has its own negative effects which in turn counteracts any benefits of the apparent slowdown. Another point arises here which requires attention. It is the possibility, indeed probability, that pace is a variable which has an optimum level such that either an increase or a decrease from it is likely to produce a reduction in learning. It is also likely true that optimum pace is unique to individuals, and hence a general increase in pace will be beneficial to some students and detrimental to others. If we return to the data now, and focus on the fact that the two countries with the lowest mean age at pop A are also high in yield at pop B. we may suggest that the pace in the elementary grades could be too slow in many countries. If this were not true then why would the apparently fast-paced nations do so well? When we move to the pace issue at the secondary level the situation is quite clearly reversed and the nations with the slowest pace through the pop B years are the ones which seem to be benefitting the most. Thus, the Cockcroft-Damerow hypothesis is most likely to be helpful in our analysis of the secondary school program.

The SIMS data provide another way of looking at the pace issue. The «opportunity-to-learn» data have been aggregated in a matrix form which reveals item-by-item and class-by-class coverage. In the case of Country E, for instance, there are five matrix displays shown in Figure 4, one for each of the item clusters arithmetic, algebra, geometry, probability and statistics and measurement. Each row corresponds to a test item in the SIMS Pool and each column corresponds to a class (Country E had a sample of 85 classes), and hence there are 85 columns in each of the matrix displays. The reason that this form of display takes on meaning in the pace debate is that the ordered columns clearly distinguish between classes with higher and lower coverage. This same distinguishing among classes on coverage is, in fact, a distinguishing among classes on the variable we have called pace. The fast-paced classes are on the left; the slow-paced on the right. My discussions with classroom teachers about these matrices has reinforced my own view that we have at present wide variations in pace among our

4 Cf. Footnote 2.

classes and that these variations are likely the result of the classroom teachers' attempt to accommodate the pace to the capabilities of the students. The fact that the matrix displays for virtually all countries reveal similar variations in pace among classes suggests that there is some common reality tied to such data. What that reality is, and how it can be utilized in moving towards a viable mathematics-for-all is the present problem. My feeling is that the reality here is precisely the one we are looking for, namely that pace can be slowed, and in fact is already slowed, for the advantage of the below average student.

The potential for benefits from manipulating pace is suggested by a consideration of some matrix displays at pop B. Figure 5 shows Country E and Country F. and the feature of these displays which is most evident at first glance is the low coverage in «analysis» for Country F compared to high coverage for Country E. If Country F should prefer to cover the same amount of analysis as Country E it could consider the possibility that the extra time in secondary school (55 months versus 47 months) could do it. Also, the possibility that lower retention could be a factor must be considered. My own speculation is that the slower pace in secondary school in Country E accounts for both higher coverage (than Country F) and for higher retention (than other countries except Country F).

There are insufficient data at the present time to warrant definitive claims, but the debate is just beginning now, and I believe the SIMS data and their various new forms of aggregation will eventually lead us to general relationships among truly fundamental educational variables. If we can indeed manipulate learning through the manipulation of pace then we may begin the process of moving towards mathematics-for-all even before we know what the nature of the content should be.

I wish to close my contribution to the debate on mathematics-for-all with a brief and simple analysis of the issue of content selection. I wish to suggest that a rationale for mathematics which appeals to the «new» clients of mathematics, i.e., the middle level students who constitute the backbone of society, must be carefully constructed. I believe that a market oriented rationale is quite appropriate. Such an orientation is likely to be widely accepted if it is true, and if it can be shown to be true, that the students in the middle and below the middle on our mathematics competence scale will be required to use mathematics, or «mathematics-for-all» in their chosen work in the marketplace. Some observers suggest that mathematical-skill-based sophistication in the marketplace will increase dramatically as hi tech moves into a dominant market position. Under such circumstances it is true that mathematical-skill-based sophistication should be introduced into the core of the mathematics curriculum which Damerow is proposing. How to identify the precise ingredients in this new core is not known, but there should be mechanisms available for arriving at a best guess.

While it is true that many observers see an increasing need for mathematical sophistication on the part of the average worker, there are other observers who suggest that the «user friendliness» of computers yet to be introduced in the marketplace will place fewer demands of a mathematical nature on the typical citizen in the workplace. I have not yet seen a clear answer for this issue. My intuition suggests dramatic increases in sophistication but I have no data to support my intuition. Perhaps mathematics educators should be prepared to collaborate with representatives of government and business in an effort to identify the generic skills needed in our new core mathematics-for-all. Then we can hope to make significant progress in the quest for mathematics-for-all.

Table 1: Implemented Coverage Indices C* (see Figure I below)

POP A		Population A MEA (14) MEAN (SD)	ARITH (46) MEAN (SD)	ALG (30) MEAN (SD)	P&S (18) MEAN (SD)	GEO (39) MEAN (SD)	WGTD MEAN (SD)
Country	A	94 (8)	85 (29)	83 (27)	75 (28)	52 (31)	75
	B	84 (12)	84 (20)	81 (23)	52 (17)	56 (27)	72
	C	79 (5)	74 (15)	72 (17)	68 (19)	69 (23)	72
	D	74 (12)	73 (17)	60 (16)	63 (17)	50 (22)	70
	E	82 (12)	86 (16)	69 (20)	61 (20)	49 (23)	69
	F	74 (14)	85 (17)	82 (15)	47 (20)	49 (27)	69
	G	88 (12)	83 (24)	83 (14)	49 (21)	42 (27)	68
	H	74 (11)	84 (18)	68 (18)	70 (19)	44 (24)	67
	I	80 (10)	77 (19)	66 (21)	32 (14)	61 (27)	63
	J	68 (15)	66 (25)	62 (23)	58 (18)	60 (30)	63
	K	71 (19)	74 (28)	69 (22)	51 (21)	37 (20)	60
	L	81 (13)	74 (25)	71 (19)	37 (22)	29 (30)	58
	M	81 (22)	79 (18)	51 (21)	32 (25)	35 (25)	56
	N	51 (8)	59 (15)	70 (12)	43 (17)	36 (20)	52
	O	64 (24)	62 (30)	42 (22)	42 (19)	32 (26)	48
	P	31 (12)	28 (16)	38 (14)	28 (13)	28 (19)	30
MEAN (SD)		74 (15)	73 (15)	67 (14)	50 (14)	46 (12)	62 (12)

Table 2: Mean Age in Months

Country	Pop A	Pop B	Diff.
A	162	217	55
B	171	218	47
C	171	217	46
D	170	217	47
E	167	222	55
F	168	215	47
G	170	214	44
K	166	223	57
L	170	217	47
N	168	214	46
O	167	228	61

Table 3: Implemented Coverage Indices C

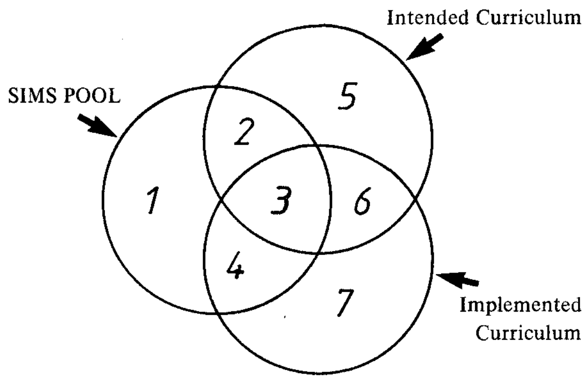
				Population B				
POP B	ALG (25) MN (SD)	ANAL (45) MN (SD)	NUM (19) MN (SD)	SETS (7) MN (SD)	FIN (4) MN (SD)	P&S (7) MN (SD)	GEO (29) MN (SD)	WGTD MEAN (SD)
Country A	100 (1)	94 (7)	82 (32)	95 (9)	99 (0)	83 (26)	85 (28)	91
E	91 (10)	92 (10)	81 (15)	83 (19)	93 (6)	72 (16)	74 (23)	85
J	92 (10)	94 (11)	88 (12)	85 (22)	52 (10)	86 (11)	68 (28)	85
K	92 (19)	88 (13)	87 (13)	87 (16)	83 (6)	85 (17)	70(37)	84
L	91 (10)	87 (13)	76 (21)	89 (11)	63 (3)	44 (16)	76 (24)	82
D	85 (12)	86 (13)	73 (18)	55 (19)	60 (6)	67 (20)	62 (30)	75
O	84 (18)	83 (18)	80 (21)	56 (28)	83 (4)	73 (16)	55 (37)	75
C	62 (12)	73 (16)	68 (14)	75 (8)	70(22)	77 (20)	62 (19)	68
H	87 (16)	57 (24)	80 (19)	81 (19)	55 (17)	46 (30)	52 (37)	65
B	70 (14)	60 (27)	69 (13)	75 (12)	85 (6)	81 (7)	53 (28)	64
N	56 (20)	73 (20)	50 (21)	28 (17)	43 (4)	17 (6)	35 (35)	52
F	81 (22)	35 (33)	74 (28)	66 (27)	9 (12)	28 (34)	43 (38)	51
MEAN	82 (13)	77 (18)	76 (10)	73 (19)	66 (25)	63 (24)	61 (14)	73 (13)

* (Not collecting OTL data were: Belgium (FR), Hong Kong, Nigeria and Scotland.)

Table 4: Coverage Retention and Yield

	Population B		
Country	Ct	R	Y
A	89	12	107
B	59		
C	65	50	325
D	71	6	43
E	84	19	160
F	49	30	147
H	65	12	78
J	84	11	72
K	84	15	126
L	84	10	84
N	46	6	28

Figure 1 : Item Classification



n_i = number of items in space i

$$C = \frac{n_3 + n_4}{n_1 + n_2 + n_3 + n_4}$$

= Coverage Index (implemented)

Figure 5 : Country E

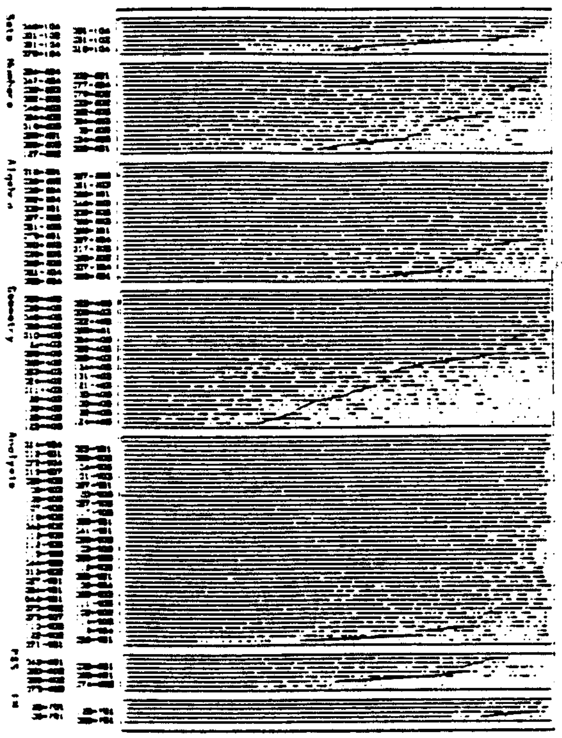


Figure 4 : Country E

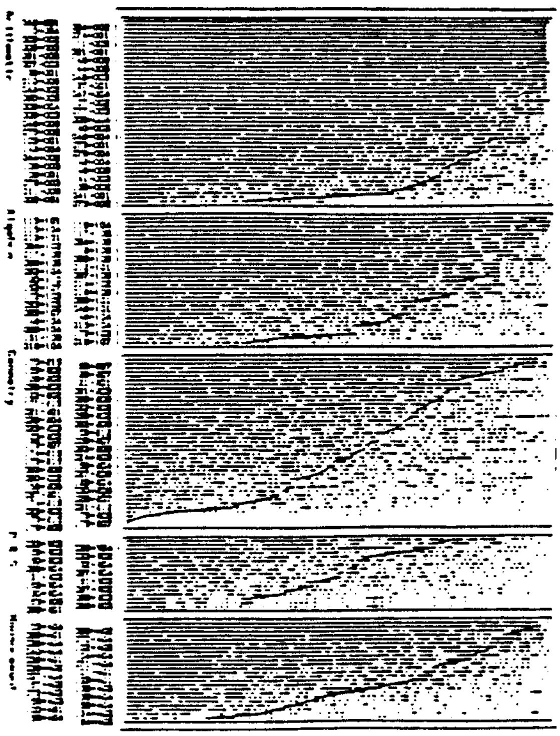
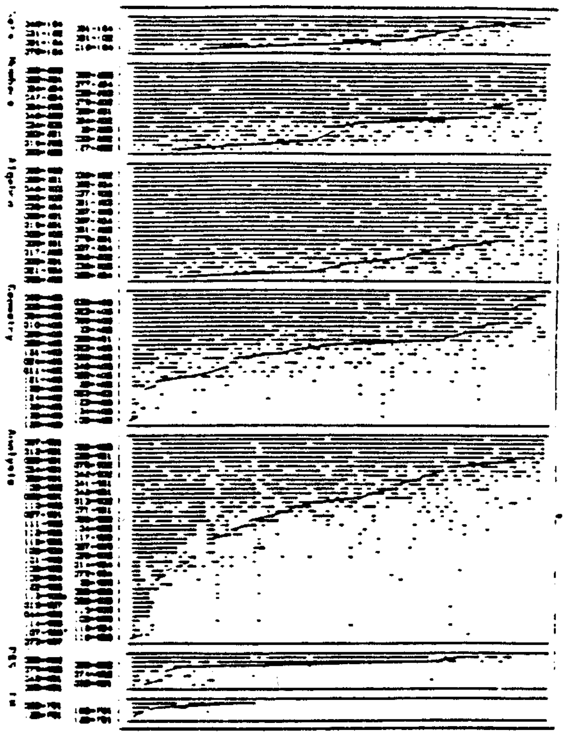


Figure 6 : Country F



Implications of the Cockcroft Report

Afzal Ahmed

1. Mathematics permeates the whole society and its use seems to assume ever increasing importance as our societies become more technological and complex. Mathematical methods and thinking are not the prerogative of scientists, engineers and technologists only, they are used by people in making everyday decisions. Their use in analysing individual behaviour, to study opinions and attitudes is also increasing. The place of mathematics in both primary and secondary school curricula for all pupils is also evidence of general agreement that the study of mathematics, along with language is regarded by most as essential. Is it not ironic that the subject which has assumed such prominence in society is also that which is most closely related to failure? Low attainment in mathematics has certainly been at the centre of the education debate in Britain throughout this century.

2. The Committee of Inquiry into the Teaching of Mathematics in Schools in England and Wales, of which I was a member, was set up in 1978 as a result of concern about the mathematical attainment of pupils.

I must be careful not even to attempt to summarise the report of this committee «Mathematics Counts», which was published in January 1982.1 I would, however, use relevant evidence from this report to discuss the aims and focus of the Curriculum Development Project for Low Attaining Pupils in Secondary Schools Mathematics which I am directing. This is a three-year project which commenced in September 1983 and is one of three projects commissioned by the Department of Education and Science as a result of the concern raised in the Cockcroft Report (The other two projects concern the assessment procedures for low attaining pupils.) I shall not confine this paper to the work of this project nor does the paper contain official views of the project. I merely use the findings of the Cockcroft Report and my own project to support the views expressed in this paper.

Paragraph 334 of the Cockcroft Report begins with the following sentence:

> Low attainment in mathematics can occur in children whose general ability is not low. «

This, of course, is true of adults too, and this fact is illustrated quite vividly in Brigid Sewell's report on the use of mathematics by adults in daily life.2 This enquiry was undertaken in association with the Cockcroft inquiry, Section 6(ii) of this report states:

> «Many of the people interviewed during this enquiry were inhibited about using mathematics, this led them to avoid it as much as possible and in some instances it has affected their careers. The inhibition was most marked among women who had specialised in arts subjects. The more educated were affected to a much greater extent than the less educated.»

3. The solution to the problems of low attainers in mathematics is not simply dividing pupils into the following three groups and then providing separate curricula for each:

a. those who are good at mathematics;
b. those whose general ability is not low but are failing at mathematics;
c. those whose general ability is low and are failing at mathematics.

The problem is much more complex since we do not know enough about the way children learn, we are not agreed on the nature of mathematics and there is little known about effective teaching methods. Moreover, the differentiates at which pupils learn and a wide variation in attainment at a particular age, make it impossible to categorise pupils in the above groupings with any degree of permanence. Another main factor is that teachers want to keep all options open to enable pupils to enter public examinations at the highest level possible. There are further factors such as previous school experience, environment, attitude and motivation which influence the attainment of pupils in mathematics, so the idea of separate maths curricula for separate groups does not offer much promise.

4. Past attempts at making suitable curriculum provision for mathematics for all pupils have focused on change of content, groupings of pupils and management of resources. The impact of these changes on mathematics education has not been significant Mathematics for the Majority.[3] The Schools Council Project in Secondary School Mathematics was set up in 1967 to help teachers construct for pupils of average and above ability. The courses relate mathematics to pupils' experience and provide them with some insight into the processes that lie behind the use of mathematics as the language of science and a source of interest in everyday life.

This was an admirable aim and the project was inspired by the «Newsom Report» published in 1963 under the title Half our Future.[4]

The following two quotes would, I hope, indicate the inspiring nature of this report:

1 Great Britain Department of Education and Science (1982): Mathematics Counts Report of the committee of Enquiry into the Teaching of Mathematics in Schools the Cockcroft Reporter HMSO.

2 Sewell, B. (1982): Use of Mathematics by Adults in Daily Life. Advisory Council for Adult and Continuing Education.

3 Mathematics for the Majority (1970): Chatto & Windus for the Schools Council.

4 Ministry of Education (1963): Half our Future. HMSO.

«Our aim in the teaching of mathematics to all pupils, to those with average and below average ability no less than to those with marked academic talent, should be to bring them to an interest in the content of mathematics itself at however modest a level.» (Paragraph 459)

«Few, if any, of our pupils are ever likely to become mathematicians, but some may well come to find satisfaction in mathematical work if its purpose has first been clearly seen and confidence established through the successful use of mathematics as a tool.» (Paragraph 422)

Note: *Our pupils* refers to half the pupils of secondary schools, i. e those for whom public examinations were not originally designed.

This project contained some very exciting, attractive and relevant material for teachers and pupils but the evidence of its lack of impact on schools is provided by the following comments from recent reports on mathematics teaching in secondary schools.

«The work was predominantly teacher controlled: teachers explained, illustrated, demonstrated, and perhaps gave notes on procedures and examples (...) A common pattern particularly with lower ability pupils was to show a few examples on the board at the start of the lesson and then set similar exercises for the pupils to work on their own (.) At the worst it became direct 'telling how'by the teacher, followed by incomprehension on the part of the pupils (...)» (H M I Secondary Survey, Chapter 7, Section 6.35)
«(. . .) and in the majority of the classrooms the teaching did not aspire to do more than prepare the pupils for examination (...)» (H. M. 1. Secondary Survey, Chapter 7, Section 6.25)

From the Cockcroft Report[6]

«(...) Mathematics lessons in secondary schools are very often not about anything. You collect like terms, or learn the law of indices, with no perception of why anyone needs to do such things. There is excessive preoccupation with a sequence of skills and quite inadequate opportunity to see the skills emerging from the solution of problems. As a consequence of this approach, school mathematics contains very little incidental information. A French lesson might well contain incidental information about France - so on across the curriculum; but in mathematics the incidental information which one might expect (current exchange and interest rates, general knowledge of climate, communications and geography, the rules and scoring systems of games; social statistics) is rarely there, because most teachers in no way see this as part of their responsibility when teaching mathematics We believe that this points out in a very succinct way the need -which is by no means confined only to courses for low-attaining pupils - to relate the content of the mathematics course to pupils experience of everyday life.» (Paragraph 462)

5. It is interesting to note that although recent reports point out that teachers seem to concentrate on teaching rigidly to examination syllabuses and spend a vast amount of time on teaching routine skills they are not successful in increasing the proportions of pupils who Can perform these skills. Interesting evidence of this is provided by Dr. Margaret Brown in her article Rules Without Reasons? «[7] According to her, in some cases the proportion actually decreases!

5 Great Britain, Department of Education and Science (1979): Aspects of Secondary Education in England. HMSO.
6 See Footnote 1.
7 Brown, M. (1982): Rules without Reasons? In: International Journal of Mathematics Education, Science and Technology. Vol. 13, No. 4, pp. 449-461.

Further serious and disturbing evidence has been quoted in Paragraph 444 of the Cockcroft Report. It points out that according to examination board regulations, the 16-year-old pupil of average ability who has applied himself to a course of study regarded by teachers of the subject as appropriate to his age, ability and aptitude may reasonably expect to secure grade 4 in the certificate of secondary education. The mark required to achieve grade 4 in mathematics is often little more than 30% ! This implies that pupils of average ability can only obtain one-third of the possible marks, and it can only damage pupils' confidence since the examinations are normally set on the syllabuses which teachers say they need to spend most of their time on.

6. So what are the reasons for teachers continuing to teach in largely ineffective methods? The reasons are complex but not unsusceptible to some analysis. In many cases teachers are not unaware of the failure of the system they are operating but their perception of the constraints which force them to operate in a restricted way is often misleading and mixed up. If one were to ask them the reasons for not changing their teaching approaches, as I often do, one is likely to receive a fairly standard catalogue of reasons such as resources, time, class size, disruptive pupils, rigid examination system, lack of pupil motivation, demands from employers and universities, pressure from parents, political pressure, lack of suitably qualified mathematics teachers, lack of technical support in mathematics departments and so on. Some of these pressures are real and others' perceived ones only but they do keep teachers locked up in operating an ineffective system.

7. The Cockcroft Report has pointed out that the mathematical education which many pupils are receiving is not satisfactory and that major changes are essential .
The major changes as far as the teaching approach is concerned are outlined in Paragraph 243, the most quoted paragraph of the report—

" Mathematics teaching at all levels should include opportunities for
- exposition by the teacher;
- discussion between teacher and pupils and between pupils themselves;
- appropriate practical work;
- consolidation and practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations;
- investigational work "

Unlike previous reports which were mainly aimed at teachers, the Cockcroft Report in Chapter 17 has recommended active co-operation of the six main agencies for effective change. Teachers at the front local education authorities providing support, examining boards, central government, teachers' training institutions and those funding curriculum development and educational research. Co-operation is also sought from employers and the public at large.

8. In this climate of support from the above agencies, with a nation-wide program of «post Cockcroft» activities supporting us, we have found that the following aims of our project for low attaining pupils in secondary schools can be realistic and help focus attention on aspects crucial in bringing about change in the teaching of mathematics:

- to encourage teachers to change their attitudes about ways in which low attainers learn mathematics;
- to help teachers to interpret the Cockcroft Committee's «Foundation Lists» (Paragraph 458) in the spirit of Paragraph 455 to 457 and 460 to 466, i.e., suggest activities which should involve low attaining pupils in a wider range of mathematics than the usual restrictive diet of «basics»;
- to provide teachers with ideas and strategies which should enable pupils to change their perceptions of mathematics, encourage them not to view the subject just as a body of knowledge which has to be «passed on» fact by fact;
- to suggest ways in which teachers can continually gain insight into pupils' mathematics without having to rely on formal tests;
- to suggest ways in which pupils can arrive at conventional methods and terminology through their participation in problem-solving activities and investigatory mathematics;
- to suggest ways of working which should enable pupils to see links between mathematics and other subject areas;
- to suggest ways of working which should help teachers to develop pupils' confidence and independence in handling mathematics;
- to suggest approaches which should help teachers cope with different rates of learning amongst low attainers.

9. We have begun with a viewpoint that there is already a large amount of material available for pupils, and the problem lies in its incorporation into the classroom. Transfer of material from one teacher to another, or one classroom to another is not straightforward and often causes sufficient difficulties for the second teacher to reject the material as unsuitable or justify its failure by cataloguing external pressures.

We have concentrated in the first instance on developing «good practice» in twelve chosen schools from six local counties. One teacher/researcher has been released from timetable commitment from each case study school on a fixed day every week. We find it essential that teacher/researchers are not released full-time and for the rest of their working week they are in the real environment of schools where changes are intended to take place. The value of this work will lie in the opportunity to find common and distinctive features, to follow these up and probe further in order to provide a basis for making decisions about any contributory reasons for success and failure of material and methods.

These case studies should enable other teachers to identify with situations and experience and to predict the likely measures of success in their own classrooms. We are also convinced that the growth in implementation of changes will not come about by written publication only. We are using a «cellular-growth» model for development and dissemination of ideas. In all the six counties we have developed a growing network of teachers, through full and part-time inservice courses, who are participating in trials and feedback. Naturally, the teacher/researchers are increasingly involving all the other teachers of mathematics in their own school, and we have considered it most important that teachers of other subjects appreciate the changes taking place and support them.

We are hoping that inservice packages (including video tapes) related to case studies will be produced for general dissemination to advisory staff, heads of departments in schools and teacher-training institutions. It is even now apparent that this will serve to outline the development process outlined above. We think it very unlikely that there is any short cut.

10. It is not my intention to discuss in detail all the areas of exploration we have undertaken so far, but only to provide a glimpse of some significant issues relevant to the theme of this paper.

In considering the courses for 11- to 16-year-old pupils the Cockcroft Report in Paragraph 451 states:

> «we believe it should be a fundamental principle that no topic should be included unless it can be developed sufficiently for it to be applied in ways which the pupils can understand (...)»

The chief reason offered by teachers for not using methods which enable pupils to apply their mathematics is the lack of time. I believe that the issues are more complex than this and are associated with confidence and the scale of perceptual leap required in changing their beliefs about how children learn mathematics.

The Cockcroft Report points out:

> «In order to present mathematics to pupils in the ways we have described it will be necessary for many teachers to make very great changes in the ways in which they work at present (...) (Paragraph 465)

Enabling these changes to take place so that teachers implement them from the position of conviction and confidence is at the centre of our project.

11. One major obstacle for teachers in changing their methods of teaching is their anxiety about «covering» the syllabus. This is further complicated by the fact that very little effort has been made to disentangle the teaching of those aspects which almost all pupils need to come across such as reading charts, diagrams and tables, interpreting simple statistical data, simple ideas of probability, ideas of inference and logical deductions, developing a feel for simple measurements, visualising simple mechanical movements and many other areas outlined in the Cockcroft Foundation List (Paragraph 458) from the teaching of those more sophisticated parts of mathematics which the able and interested pupils might study, e. g. deductive geometry, calculus, algebraic manipulation etc. The able minority often misses out

on the aspects of mathematics for all outlined earlier since these are glossed over by their teachers who regard these topics not as an area of experience but as «bits» of knowledge pupils need to possess.

Teachers of mathematics, who themselves were mainly an able minority, as well as pupils tend to resort to the way they were taught mathematics. There is little change in the methods used to teach the aspects of mathematics which the able minority will learn compared with those aspects covered by most pupils (including the able). The main change consists in diluting, or what is referred to as «watering down» the content and presenting topics in small easy steps with plenty of practice examples. The result is trivialising and makes mathematics meaningless for most pupils. A large number of pupils, even those are able, tend to lose interest and find little meaning in this activity and hence lose confidence in the subject. There is also a tendency for teachers to teach topics on a syllabus, systematically, item by item and believe that if all the topics have not been taught their pupils will suffer.

In order to ensure that topics are covered, it is possible to rush these through in a narrow, restricted manner rather than embedding them in a wider context allowing pupils to reflect upon the use of the mathematics presented to them and discussing the appropriateness of methods used by them. Algorithms are often taught to pupils too soon, and there seems to be an assumption that once taught they are remembered. The Chelsea Report, Understanding Mathematics 11 - 168 points out:

«The teaching of algorithms when the child does not understand may be positively harmful in that what the child sees the teacher doing is 'magic' and entirely 'divorced' from problem solving.» (Chapter 14)

Mathematics, in daily life, is not encountered in small packages as taught by fragmenting a syllabus or in the form of the straight-forward command of the textbooks. It appears in context in a variety of spoken or written language and social situations. The Cockcroft Report proposes that its Foundation List of Mathematical Topics (Paragraph 455) should form a part of the mathematics syllabus for all pupils. This list reflects situations in which people meet mathematics in life, and the emphasis of the report is on presenting mathematics in a context in which it will be applied to solving problems.

12. «At all stages pupils should be encouraged to discuss and justify the methods which they use.» (Cockcroft Report, Paragraph 458)

The implications of the suggested changes in teaching styles for teachers are great for those who have not worked in this manner. It would certainly call in question the conventional method of designing curriculum in terms of topics, concepts and skills. For example, it could mean that a teacher would need to have a collection of «situations», problems and investigations to offer to pupils and the consideration of mathematical outcome of these in terms of content and processes for individual pupils would be a retrospective activity by the teacher. For example, consider a situation where a pupil is presented with 5 pence and 7 pence stamps and asked:

a. What totals can he make if he has as many of each as he wishes?
b. Are there more?
c. Can he convince others?
d. What happens with other stamps,
   3 pence and 5 pence?
   5 pence and 9 pence?

Some of the outcomes of this activity, depending on the levels at which individuals tackle this problem, could be listed as follows:

Generalisation and Organisation of work Pattern spotting	Factors Multiples Addition and multiplication
Hypothesising and testing Formalising Exclusion Checking etc.	Algebra Permutation Combinations Primes and composites Triangles Modular arithmetic etc.

For a confident and experienced teacher, this way of working could enable him to overcome the major problem of not limiting a child's attainment since the choice of rich starters would enable all pupils to become engaged in the activity and offer an opportunity for follow-up work in depth for those who were interested and able to do so: This would certainly help to overcome the problem of offering motivating, relevant and challenging mathematics without restricting pupils' opportunities for taking any external examinations. Although the change in emphasis may appear small, the solution is not as simple as it appears. Teachers who are used to assuming the major control of their pupils' learning find it extremely difficult to change their focus. Pupils also become used to the idea that teachers will always have the right answers and the right method for all problems, and if they wait long enough these answers would be provided by the teacher directly or through the «bright» pupils in their group. Under these circumstances, pupils do not find it easy to change their role and assume responsibility for their own learning.

These changed perceptions of mathematics learning and teaching need to develop in a climate of mutual trust and confidence.

13. In her publication «Generating Mathematical Activity in the Classroom»[9] Marion Bird has used written records of a class of 11-year-old pupils to demonstrate that it is possible to teach in a way which encourages pupils to begin to ask their own

8 Hart, K. M. et al. (1981): Children's Understanding of Mathematics 11-16. John Murray.

9 Bird, M. (February 1983): Generating Mathematical Activity in the classroom. west Sussex Institute of Higher Education.

questions, to control the direction of their investigations, to make conjectures and think how to test them to make and agree on definitions and equivalences, to search for patterns, to make generalisations and to seek for reasons for what seems to be happening.

One of our project activities has been to examine a large number of case studies developed by teachers who have been trying out these teaching approaches in the classroom and identifying general features which facilitate the work and the mathematical activities and those features which inhibit them. These «facilitators» and «inhibitors» have been found very helpful by teachers attempting such an approach in the classroom. We have been exploring methods of involving teachers of other subjects in supporting these developments since their attitude can have a significant influence on pupils. We have also been exploring simple but effective methods of sustaining networks of teachers who can offer each other support, encouragement and stimulus - all important ingredients of bringing about effective change. A collection of ideas and strategies which would enable teachers to initiate a greater change of focus in the control of learning in the classroom has been compiled. This has also provided some initial starting strategies which have been tried out in several classrooms.

We are also observing and developing case studies on effectiveness of various strategies for inservice support, e.g. teachers visiting each others schools, teachers released to work with other teachers in their own schools etc.

Finally, it seems clear to us that the most effective change can come about by starting from the teachers' own strengths and building from there. This, in the time of great pressure on inservice finance, can be an extremely slow process. It would be counterproductive and a retrograde step if we allowed ourselves to be seduced by short cuts which entail offering crutches to support teachers.

The development of the Mathematical Association Diploma in the Teaching of Mathematics to Low Attaining Pupils [10], for which some teachers can be released for one day a week over a year under a central government scheme is already proving an effective agent for change. We need to think seriously about the continuation of these developments undertaken by teachers who have completed these courses and widening the means of supporting those who are not able to have such an opportunity. The extreme importance and enormous benefits of teachers released from schools to work with other teachers and the effective programs for such activities can be a theme for another paper!

10»The Mathematical Association Diploma in the Teaching of Mathematics to Low Attaining Pupils in Secondary Schools» was piloted at the West Sussex Institute of Higher Education, the Mathematics Education Centre and at Bishop Grosseteste College Lincoln.

Details available from:
The Secretary of the Diploma Board
The Mathematical Association
259 London Road
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England

Universal Mathematics Education and its Conditions in Social Interaction

Achmad Arifin

As far as I understand, universal mathematics education is mathematics education having everyone in a society as its target. Its objective is to educate the society to be more intelligent in utilising the available resources or opportunities to improve their welfare and prosperity. What we need is a mechanism to bring mathematics education to everyone in the society. This mechanism should have the ability to disseminate the intended changes or improvements at any time. School system should be in the mechanism. But the school system alone cannot be made responsible for bringing mathematics education to everyone or to carry out improvements in mathematics education.

Improvement followed by another improvement, or change followed by another change, will be the feature of mathematics education in particular as a consequence of the rapid development of mathematics as well as that of science and technology.

The mechanism should be able to channel the needed changes and improvements to the school system without any disturbances, that is to say, as smooth as possible. For a country with a large population the need in such mechanism can be very urgent.

In the implementation of universal mathematics education the active participation of the community can contribute a great deal in reaching the target.

In this paper I will try to describe how we should raise community participation in carrying out universal mathematics education through looking at the aspect of interaction. In developing the mechanism the mathematics itself and its process of development should be brought as close as possible to the place where mathematics education is going to be developed. Because the role of the mathematicians and their activities is also significant in initiating the development of universal mathematics education.

1. Social Structure

Setting the ultimate objective of education as to develop the society toward attaining a better standard of living means that education should be the concern of everyone in the society. To enable the society or the community to participate in the process of education a certain interrelationship has to be developed between the society and the school system. Since interaction is a component which basically supports the educational process, we may examine the interrelationship mentioned above from the aspect of interaction. To pave our way to this purpose, we generalise our examination by looking at the interactions that happen in a society.

Social structure in a society is understood to be the totality of interactions among people or groups of people. Depending on its quality, social structure can influence the survival and the development of the society. Through interactions the society improves its ability for continuing its survival by utilising the resources and opportunities that are available in its disposal. These interactions are social interactions.

Let us direct our attention to the social interactions which contribute to the improvement of people's ability and refer to this particular interaction as a positive interaction. Since interaction can happen between people and their environment, social as well as natural one, we generalise the meaning of positive interaction as to include not just the social one.

2. Positive Interaction

Looking at a particular society we may always ask whether interactions happen among individuals in that society. Assuming that interactions happen among them we may further ask, what are the kinds of interactions and how intensive they are. What we should identify are those interactions which have the effect on increasing individual's knowledge or skill, or individual's ability in general.

On the other hand, someone may gain additional knowledge or skill through reading books or observing natural phenomena. Can we say that someone gains additional knowledge and skill through interaction between him or her and the books he or she is reading or between him or her and the natural phenomena he or she is observing? The answer is yes, but this will depend on the way he or she reads the books and observes the phenomena. Certainly someone needs a certain ability in order to undergo these kinds of interactions.

We generalise the meaning of positive interaction to include any interactions which contribute to the development of abilities of individuals or groups of individuals. It can happen in various kinds and patterns between individuals and their surroundings. This positive interaction is actually an important aspect in the educational process. It provides opportunities to individuals to improve their ability. With their continually improving ability the people will have opportunities to contribute to the development of their social structure.

Positive interaction is expected to happen life-long for each individual. Someone needs some knowledge and skill to initiate positive interaction with his or her surroundings. In particular, someone should be able to utilise information to get some additional knowledge and skill, or to improve his or her ability in general. Facilities like libraries or museums which exhibit new developments, for instance in science and technology, form resources of information. These can provide stimuli to motivate positive interaction to happen continuously with increasing intensity and quality as to fulfill the needed ability.

Someone's continual efforts for improving abilities, or developing new abilities can be considered as the consequences of the rapid development in various sectors. The developments in science and technology in particular create challenges in their utilisation. To cope with these challenges, particular abilities have to be made to exist. Leaving the development of such needed abilities to individuals' motivation and initiation, through their involvement in positive interaction, might take a long time. Therefore, an organised effort is needed to design positive interaction to happen in a certain time period and space.

3. School Interaction

Someone needs to have certain abilities in order to be able to get involved in a positive interaction. Someone needs to know the language of the book he is reading, or perhaps he needs some knowledge about the topics discussed in the book in order to get some additional knowledge about the topics. In general, someone needs some basic knowledge and skill to draw additional knowledge from the book he is reading, from the discussion with someone else, from the observation of a natural phenomenon, etc.

To organise positive interaction to happen in a certain time period and space is particularly aimed at providing individuals with some basic knowledge and skill. Positive interaction which is designed to happen in a certain time period and space is an important aspect pertinent to what we understand as school. I refer to this positive interaction as school interaction or classroom interaction. The ability of individuals in carrying out further development of their own ability as well as that of their society independently is usually set as the main objective of school interaction. This further development of ability can be carried out through positive interaction.

A school is a place where people who have different backgrounds and come from different environments come together with the intention to learn. The people who come to learn at a certain school constitute a society of students with a certain characteristic, that is a certain level of readiness to get involved in positive interactions.

On the other hand, the school is equipped with a curriculum containing a series of programs of teaching and learning process as a means to achieve the objective as set in the curriculum. In this junction where the interface between student background and school curriculum take place we expect the teachers to play their role; that is to manage the school interaction to happen as to give an optimal result.

In managing school interaction we should pay attention to the individual's background as differences related to value systems or behaviour patterns might occur. Referring to those differences the teachers or someone else who acts as a facilitator should manage the interaction so as to happen without disturbances.

Each of the three components: social structure, positive interaction and school interaction should have the ability to absorb some developments in mathematics that are relevant to the society development and to utilise them in educating the people in accordance with their respective role.

The parts of each component include:

- Social structure: To appreciate and support necessary changes in school mathematics and create conducive environment outside school for learning mathematics.
- Positive interaction: To set up facilities other than schools that motivate and create opportunities for mathematics learning.
- School interaction: To develop the environment in school and the teaching methodology in mathematics class as well as that for individual approach so to enable them to inspire, to stimulate and to direct learning activities.

Referring to the roles of the components as described above, we are not looking at them with their passive meaning; but with their active participation in providing opportunities and stimulation for mathematics learning. This justifies the purpose of the three-component functional relationship: social structure - positive interaction - school interaction, that is to provide opportunities and stimulation for mathematics learning through interaction, aimed at everyone in the society.

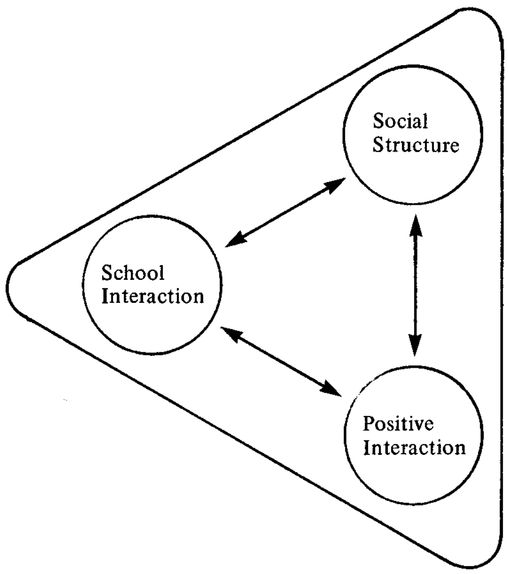
4. MathematicsforALL

Once again, universal mathematics education is mathematics education for everyone in a society, so it is mathematics education for all. The school system is one of the places where universal mathematics education can be channeled to reach a part of the members of a society. In other words, school interaction is one of the components for reaching individuals in a society and to manage them to get involved in learning mathematics. Since learning mathematics at school is related to the development of a society as a whole we also have to examine the components which can contribute in paving the way to make the relationship really happen. These components are social structure and positive interaction.

What parts of mathematics that a person learns at school should be accepted and appreciated by the society as a part of the society development process. On the other hand, the school should be knowledgeable and be aware of the needs of the society, particularly those concerning the society development and those related to increasing the individual's ability.

From now on, we need to attach more operational meaning to positive interaction. That is, some facilities need to be created or to be established in order to stimulate and provide opportunities and means for positive interaction to happen. In this connection, the targets are the society and the school system.

From what has been explained above, we can see that the interrelation among social structure, positive interaction and school interaction enable the development of the school system as well as the quality and the intensity of social interaction continually. This will result in increasing the ability of individuals. Furthermore, referring to the role of individuals in increasing the quality and the intensity of interaction the continual development of school mathematics teaching will, after all, result in increasing the ability of the society itself. Therefore, taking also into consideration the individual's background during teaching-learning processes we can describe a functional relationship among social structure, positive interaction and school interaction as follows.



This three-component functional relationship forms a mechanism for developing the needed ability of a society in coping with challenges. Therefore, it can also be utilised as a means of transferring mathematics and its development to everyone in the society, through various kinds and patterns of interaction.

This is aimed at achieving certain mathematical abilities. This gives an idea as to how to carry out universal mathematics education. To initiate universal mathematics education it is therefore desirable that in any society the three-component functional relationship can be identified.

In this era of science and technology development, the intelligence of the people will contribute a great deal and meaningfully to the society's ability. Mathematics education can play a role in developing the intelligence of the people. But this will depend on the persons who have to make opportunities to enable mathematics to play its role fully; that is to create a conducive environment where mathematics learning can happen through interaction. Referring to this, we are dealing with the following questions.

1. Which part of mathematics that can function as a developer of an individual's intelligence?
2. Those parts of mathematics that have been chosen, how should they be presented?

To learn mathematics through interaction can take place everywhere and any time, not necessarily in a classroom during a mathematics class. Therefore, those two questions are relevant to the school interaction as well as to the other two components: social structure and positive interaction.

Referring to the purpose of developing a society, we should apply the two questions to the three components: social structure, positive interaction and school interaction equally balanced. The answer will depend much on the existing cultural condition, for example, the average mathematical knowledge possessed by the people in the society, the intensity and the quality of interaction that constitute the social structure. Furthermore, the existence of mathematicians and their activities related to mathematics should provide some assets in seeking answers to the questions.

In developing countries mathematics for all can be earmarked as universal mathematics education, that is mathematics education for everybody. The mechanism for carrying out universal mathematics education is the three-component functional relationship. In developing universal mathematics education it is always necessary that we examine the mechanism whether it is an appropriate condition to carry out the new changes. Otherwise, we have to develop the mechanism itself, that is to develop the components or the interrelation among the components.

In developing countries the problems related to the development of universal mathematics education always are concerned with the development of the mechanism besides those of mathematics and its teaching. In developing countries with a large population, the problem will mainly concern the development of the mechanism, to enable it to function accordingly. This means that in carrying out universal mathematics education we have to pay attention to the three-component functional relationship: social structure — positive interaction —school interaction. Its development includes the development of the components and their interrelationship.

The role of the mathematicians from the country concerned is to form a pool of expertise for the development of the mathematical content and the way it should be presented. This includes the development of the teaching methodologies as well as that of the mechanism, that is the three-component functional relationship. Therefore, the development of universal mathematics education will never be free from the need to develop mathematical activities in the developing country concerned and the willingness of the mathematicians to take part in the efforts to improve the intelligence of their people.

5. The Role of Mathematicians

The two questions related to what and how as mentioned previously are: Which part of mathematics that can function as a developer of an individual's intelligence and how should they be presented, always occur during the development of universal mathematics education. The development of mathematics provides us with many choices in fulfilling the mathematical content, while the development in technology provides us with many alternatives for presenting certain mathematical topics.

The questions related to what and how will always be relevant from time to time. Therefore, what is more important here is the availability of a mechanism which is able to provide answers to the questions of what and how in accordance with the needs and conditions of the developing society or nation. The mechanism should include the three-component functional relationship which carries out the universal mathematics education and the local mathematicians who provide information and expertise for the development of the mathematical content.

On the other hand, the local mathematicians should be actively involved in stimulating and upgrading the three components that are aimed at developing the three-component functional relationship.

To fulfil this task the mathematicians need to keep themselves informed on the development of mathematics and actively maintain their communication with mathematical activities as wide as possible.

6. Conclusion

Referring to the role of mathematics education in developing individuals' intelligence that is aimed at developing the ability of a society or nation, the role of mathematicians in providing information and expertise, the expected active involvement of mathematicians in the development of mathematics education, the fact that mathematics and also science and technology are developing rapidly, the fact that the growth of education depends on the cultural condition of the society, we may close this paper with the following remarks:

1. Universal mathematics education can function as a means for increasing the ability of a society or a nation through developing individuals' intelligence.
2. Universal mathematics education needs to undergo continual development to maintain its function in a developing society or nation.
3. The three-component functional relationship can function as a mechanism for universal mathematics education to reach everyone in a society through the school system as well as outside the school system.
4. The local mathematicians and their mathematical activities could be supportive to the continual development of universal mathematics education including the three-component functional relationship as its mechanism.
5. The development of the three-component functional relationship as a mechanism and the supportive attitude and activities of the local mathematician reflect the effectiveness of the community participation.

Alternative Mathematics Programs

Andrew J. C. Begg

Introduction

Whenever a group of high school teachers discuss their mathematics programs they make certain assumptions. They usually take for granted the way the programs have developed over recent years, the available resources, and the purpose of the programs. In this paper I want to suggest a number of questions that may cause us to question our syllabi and teaching methods and consider alternative programs. Before looking at these questions we need to consider some aims, assumptions, and constraints that we probably all share.

Aims

In mathematics education the three most common aims of our programs are summed up as:

Personal — to help students solve the everyday problems of adult life;

Vocational — to give a foundation upon which a range of specialised skills can be built;

Humanistic — to show mathematics as part of our cultural heritage.

These three aims imply that basics are necessary but not sufficient, that we must present a broad-based course as the future needs of our students will vary tremendously, that historical topics and career information should be included, and mathematics should be given a warm and human flavour rather than a formal or logical one.

Assumptions

In my country we assume that universal secondary education, including mathematics for at least two years, and usually for three or four years, is the right of all students. This assumption varies in other countries and may make some considerations less relevant.

The other assumption I would make is that for many students mathematics is not inherently interesting — indeed they may have been turned off the subject. Usefulness is not enough, we want motivation and fun, and our mathematics programs must build in this motivation so that students look forward to our classes.

Other Constraints

A huge variety of schools exist: large/small, urban/ rural, traditional/alternative, wealthy/poor, academic/ technical etc. In spite of this variety many places, including New Zealand, have a national system of education that has one syllabus for each year of schooling.

Our yearly programs are based on 3 or 4 hours week of mathematics. In New Zealand this used to be taught in 5 x 40 minute periods but is now usually 3 x 60 minute periods. This change should cause significant movements in the teaching of our mathematics program.

The third constraint is caused by the pressures on our school syllabi. We are asked to cover numerous new items. Computer education, careers education, outdoor education, health education, and multicultural education are examples of areas that either require time from the total school program or need to be integrated across the curricula and this includes the mathematics curricula.

General or Special Purpose Mathematics

It is usually possible to look at a group of students and see some aim they have in mind, e. g. to graduate from high school, to pass an external exam, or to prepare for employment or unemployment. When we look more closely we can usually isolate a number of subsets of students with differing needs. In the same class we find students who are terminating their mathematics education, others who hope to pass an exam and others who expect to take the subject on to a higher stage.

In large schools it may be possible to separate these students into different classes which cope with their specific needs, but in small schools this is not possible. Further, the students may not be certain about their future plans and needs.

In New Zealand I find that many classes contain students requiring enrichment while others need remedial assistance, some are sitting one exam while others are sitting another, some expect to leave school for a job, others for unemployment; yet generally students are not given alternative programs to cater for these needs. At the most senior level in New Zealand we are looking at alternative mathematics programs with a statistics or calculus bias according to the student's needs, but for the majority of school leavers no basic course exists with significant elements of budgeting, tax, insurance, rents and the other skills needed by school leavers. I believe alternative programs are needed where content decisions are based on the needs of the participating students.

Teaching: Mathematics or Students

A mathematics program is part of a total educational package. As teachers and program designers we must consider the aims of this whole package and then adjust our programs to suit. These general aims would include the development of

- self-respect,
- concern for others,
- urge to enquire

and we would want to develop the skills of - communication,

- responsibility,
- criticism,
- cooperation.

If we wish to achieve some of these aims we must stress cooperative learning, encourage project work and displays of work by individuals and groups. We must use discovery approaches and not try to shortcut this time-consuming process by presenting results too quickly. We must give all students success in their eyes, in their peers' eyes and in their teachers' eyes. Here alternative programs are needed as our present ones are not achieving these goals, and we must consider other teaching methods as an important part of these programs.

Monocultural or Multicultural

Traditionally, our programs, both contents and methods, reflect the traditions of European education. Many of us have assumed this European background and the associated attitudes to learning. Now we find that mathematics programs are required for monocultural groups that are not European and for multicultural groups that reflect a broad range of cultures.

In New Zealand the two main non-European groups are Maoris and Pacific Islanders. Practically no research has been done on their different views and attitudes to mathematics, nor to the way their language reflects different views of the subject. What we are aware of is that many Polynesians prefer group work and do not enjoy being ranked apart from their peers. This fact has obvious implications in designing a program that stresses group rather than individual success.

Other problems experienced by groups from other cultures and in particular by new immigrants are the problems associated with language. If teachers are aware of these difficulties and modify their programs these difficulties can be reduced, but it is difficult for a teacher to cope when a very broad range of backgrounds occur at the same time.

In building self-esteem we must at least build respect and understanding for the differences that our students reflect. We can at least use names and subjects from other cultures in our examples, but we must be careful not to offend. Assuming this cultural sensitivity, opportunities exist in numerous areas to incorporate ideas from other cultures and help students of all races appreciate the differences between members of their communities. This two-way appreciation helps our students «stand tall».

Streamed or Mixed Ability

Disregarding all other differences and factors affecting the achievement of our students we all accept that students of a particular age group vary from very talented to those of low ability. One way of handling this variation is streaming. The difficulty with streaming (or even broad-banding) is that it is practically impossible to stream exactly, and when one considers the factors affecting achievement (illness, schoolchange, bad teaching etc.) one realises that streaming can never be perfect.

Many teachers believe that because of social factors it is desirable to keep mixed-ability forms. Certainly mixed-ability forms mean we must offer alternative programs within our classroom where with streamed groups it is easy to think one program is suitable for all students when in fact it is only suitable for the majority of them.

In smaller schools and in schools where option structures affect mathematics we are forced to cope with mixed-ability classes and to design alternatives within the program. These alternatives include enrichment and extension for the more able students and more practice in basic skills for the less able. These alternative programs need to consider the appropriateness of topics according to whether or not students are ready for the subject and whether or not they expect to continue their mathematics education into the future.

Class, Individual, or Group Programs

Traditionally, most of us taught our classes as one group. More recently, with the advent of mixedability classes, some of us have tried individual programs. Having tried both methods, I would believe that neither a class approach nor an individual program is satisfactory if used all the time, so again I look for alternatives.

Analysing the possibilities we have six main teaching modes:

Teaching Mode	Work	Rate	
Class teaching	Same	Same	
Individual program		Same	Different
Individual program		Same	Different
Group teaching	Same	Different (between groups)	
Group teaching	Different		Different (between groups)
Group teaching	Different		Different (within groups)

No approach is necessarily correct and variation of style throughout the program is probably the most desirable answer but within this varied program I think we need to use groups to a much greater extent than at present. Sometimes groups will be doing the same work at different speeds, sometimes the same work at different speeds but achieving the work at different levels, and sometimes different work. All these modes need consideration in our programs and obviously our decisions will affect the resources we need for these modes of teaching.

Cooperative or Competitive Learning

I have mentioned the way some cultural groups prefer working in groups and how using groups may help overcome some problems associated with aspects of whole-class and individual programs. I want to suggest that a group approach is the real-life approach to many problem-solving situations and that our programs should reflect this cooperative and realistic approach.

I know that we all value excellence and feel that the student who ranks first in our class deserves praise and reward, but I believe that this should not be achieved using a competitive strategy which recognises one at the expense of ranking others in lower positions. It can be done with comments rather than with numbers. At the same time, other students also do well in presentation or in attitude while others improve in some facet of their work, and these aspects need positive encouragement and praise too.

When a cooperative group is working together (whether it is mixed ability, streamed, special needs or whatever) then the whole group should share the praise and encouragement, and there is no need to separate out individual skills to achieve our objectives.

Traditional or Alternative Teaching Styles

The teaching style of many of us still reflects the style of many years ago, the way we were taught and even the lecture/tutorial system of traditional universities. Meanwhile, our clients have changed, they represent a broader range of backgrounds and abilities, their interest in mathematics is different, and they are more sophisticated. Some teachers have altered their teaching styles and some have tried discovery approaches. The new maths revolution had some teachers trying more formal approaches and others more intuitive approaches.

I believe that, in terms of our general aims, we need to encourage open-ended approaches whenever appropriate, through discovery learning and through project activities. The important aspect here is that a variety of approaches is needed, and it is often useful to have one group doing projects while another group requires more teacher attention.

We sometimes talk of integrating mathematics with other subjects in the curriculum. Science, economics, geography, technical drawing and home economics are examples of subjects where obvious overlaps exist. From our knowledge of transfer of training we must assume that subject overlap must be made explicit and that when we talk of applications in mathematics, surely the most relevant applications are those ones which the students are at present involved in with their other class work. This change must be reflected in our programs and will vary according to the other option choices of our students.

A final aspect of teaching style that worries me in New Zealand is a result of the change in school structures from 7 x 40 minute periods each day to 5 x 60 minute periods. In mathematics this has usually meant 3 x 60 minute periods replacing 5 x 40 minute periods. At the same time, many teachers have not significantly adjusted their teaching style. I believe this 60 minutes should be split into at least 4 shorter periods with a greater variety of activities occurring during the hour. Maintenance should be built in, the range of activity should include not only oral and written work, but also physical activity and a greater use of visuals both in teaching and summarising. I believe this variation should be built into our programs in recognition of the short attention span of many young students. The need to provide varied and interesting stimulation over an extended period is the only way we can avoid having our students bored.

Text-oriented or Multi-media

Textbooks are the most commonly used resource in our classrooms. They have usually been written for a particular course and are the cheapest available resource. With the advent of alternative programs we see the need to build up numerous supplementary resources. With groups working on different topics in the same classroom and with students with reading difficulties, we see other reasons for more resources. Mathcards and worksheets are needed to direct students into alternative activities such as games for maintenance work, project starters, and enrichment. Much of the work on these cards could be self-motivating. Computer-assisted learning has obvious applications to remedial, revision and enrichment work. Films, slides and videos all have their place in providing variation. Our mathematics programs must «build in» these resources so that within a school all the students are getting these advantages and not merely those in the classes of one or two keen teachers.

Imaginative texts are still necessary, and we must remember that every student should learn to learn from a book without assistance.

Logical or Humanistic Approach

Mathematics was taught very formally, then with the «new math» we saw logical approaches, mathematical approaches and psychological approaches. Some people have tried more intuitive approaches, and I understand one or two have used an historical approach.

What I would prefer is a humanistic approach, I mean an approach that is student-centred and develops from the students' particular interests and needs. An approach that links their work to real life and to applications that are relevant to them.

I want to see a warm approach that treats every student as someone special, that works positively to avoid sex or race stereotyping, and that builds self-esteem in our students. I am sure that once this self-esteem is present, teachers will be amazed at the progress students can make.

Conclusion

I know that schools have limited resources, that teachers have limited time, and that numerous other constraints are put on us by our schools, but I believe we can all introduce more alternative elements into our programs. I know most of us like to have a class start together, but we can still produce various endpoints, we can use group work, and we can encourage more cooperative problem solving.

I am sure we must give students the opportunity to make decisions that are relevant to their education and each of us should be «the guide on the side not the sage on the stage».

Program development will keep happening, it is our responsibility to make sure it helps our students achieve the aims of education in general as well as the aims of mathematics education.

Part II:
Problems and Developments in Industrialised Countries

Arithmetic Pedagogy at the Beginning of the School System of Japan

Genichi Matsubara and Zennosuke Kusumoto

In Japan the Emperor had held the reins of government, although the Samurai held it for some time in Japanese history. In the Meiji revolution in 1868, the Emperor took the reins of government again and the new government in the Meiji Era was born. To make the nation modernise, it made great efforts to learn a great deal from foreign countries and to adopt new educational policies.
About 110 years have passed since the elementary school system was established in Japan. Here I'd like to tell something about arithmetic pedagogy at the beginning of the Meiji Era in Japan.
Today everyone thinks it is the turning point in education in Japan, so that to reflect on arithmetic pedagogy in the Meiji Era will provide some suggestions on how to introduce a new arithmetic pedagogy.

I. Arithmetic Pedagogy at the Beginning of the Meiji Era

First, I'd like to discuss the «Terakoya» (private schools) in the age of the Samurai, since they played an important role in forming the basis of arithmetic pedagogy at the beginning of the Meiji Era.

A. Arithmetic in «Terakoya»
At the end of the Edo Era, the Samurai took the reins of government in Japan. In order to educate the Samurai's children, each feudal clan had a school of its own. But they did not take into consideration the education for ordinary people. So they established their own schools to get a minimum knowledge to go into the world—these were called «Terakoya». There were many «Terakoyas» but there was no communication in terms of teaching methodologies among them. Arithmetic in «Terakoya» was mostly concerned with how to handle abacuses. Many kinds of textbooks used at «Terakoya» were published in the Edo Era. Among these publications, about 1,126 books were mathematics books such as UJinkoki-, «Sanpo zukai». These were used as textbooks for children and teachers at «Terakoya» or other private schools. Next I'll introduce to you something about «Jinkoki» to let you know the contents of arithmetic pedagogy in those days.

B. «Jinkoki»
This is the book on abacuses written by the mathematician Mitsuyoshi Yoshida in 1627. This was modelled after «Sampotoshu» which was a manual for abacuses in China. But the contents mainly consisted of how to use abacuses for business transactions in daily life. It is noticeable that what was dealt with in it was just the same as what we dealt with in problem solving, which was discussed all over the world in the early 20th century. But teaching methods were not dealt with in it. After many teachers of «Terakoya» used it, the contents were revised and published several times.

C. «Terakoyosho «
The content of arithmetic at that time was to learn the four operations by using the abacus. The teaching materials connected with daily life transactions were arranged in the same way as in problem solving. There were many children who were able to do division of two-digit numbers. Graduating from «Terakoya», they went to another private school where they studied the latter half of «Jinkoki». Teachers of «Terakoya» taught them according to their own philosophy. So, when the government introduced school systems in education and build elementary schools, «Terakoya» was one of the models in mathematical education. In December 1873, Director David Murray reported to the Ministry of Education that the average standard of education was very high.

II. Arithmetic Pedagogy After the Proclamation of School Systems

After the revolution of the Meiji Era, the Samurai reign was replaced by the Tenno (Emperor) reign. Then the unified Meiji government was born. One of the policies of the government was to build elementary schools all over Japan and to put an emphasis on the 3R's. Prior to this, there were already a few private elementary schools established. But feudal clan «Terakoya» and other private schools exerted a great influence to make the new educational policies possible.
In March 1869, the government ordered the building of elementary schools in every prefecture. The government promoted the new educational policy, so people were eager to equalise ordinary education. After the Samurai Era passed on to the Tenno reign, each local government took over the policies. The curriculum of each school, similar to that of «Terakoya», was as follows: The official age of enrollment was five, but it was usually six. Almost all children could understand basic addition and subtraction. In Tokyo «Terakoya» was not admitted as a school, but later was admitted as a private elementary school. In those days the government authorised two kinds of schools.
(1) One was a school for people going into business after graduation, which was established in each prefecture. By establishing these schools, the government aimed at the decentralisation of education.
(2) The other was a school for people going on to college and the university after graduation. Then there were three kinds of elementary schools considered from other perspectives.

a. the national school — established by the Ministry of Education; the attached school - established as the preparatory course for colleges and universities; b. b. elementary schools of each prefecture — established by private funds;
c. the elementary school — established by feudal clans.

III. Proclamation of School Systems

A. Establishment of the Ministry of Education and Proclamation of School Systems

When the Meiji government started, much thought was given to educational policies. It established a kind of school administration section in the government of Kyoto. Then the capital was moved to Tokyo and the same section was established in Tokyo. So, it was thought necessary to establish the Ministry of Education, which deals with educational matters such as making rules, building schools and assisting financing. In September 1871, the government decided to abolish schools for the Samurai and to establish the Ministry of Education.

The Ministry of Education had the task of making rules about elementary schools and junior high schools and did everything for schools. It made efforts to equalise the standard of education of every school in Japan. This made it reinforce school systems in Japan.

In Japan the law on school systems was the first one that was established for education. It controlled the systems and the curricula of schools from elementary level to university level in Japan. To make the law, the government not only gathered the materials from other countries, but also established pilot schools to get data from them.

In August 1872, the outline of school systems was decreed as follows:

a. the necessity of schools in terms of man's development;
b. to study regardless of positions and sex;
c. to explain the mistakes of traditional learning;
d. to give everyone opportunities of learning;
e. to make parents responsible for children's education;
f. to make parents pay money for children's education;

The contents which children had to study were arithmetic or abacuses. We can find in «Gakumon no susume» *(Encouragement of Study* written by Yukichi Fukuzawa), how to learn the angular style of the Japanese, how to write Chinese characters and to drill using abacuses, to deal with the balance etc. were indicated.

Now I stall summarise the school systems.

1) The large, middle and small districts

The Ministry of Education, which was responsible for controlling and managing schools in Japan, divided all areas as follows:

The large districts — the country was divided into eight. A university was established in each large district.

The middle districts — each large district was divided into 32. A junior high school was established in each middle district.

The small districts — each middle district was divided into 210. A elementary school was established in each small district.

So, there were 6,720 elementary schools in one large district and 53,760 elementary schools in all.

2) Schools

There were three kinds of schools - universities, junior high schools and elementary schools.

The curricula were prepared in a special book.

3) Elementary schools

Elementary schools were considered to be the primary stage of school education, so all the people had to go to school by all means. Schools were generally called elementary schools, but there were several kinds.

a. Infant schools - preschools for children under six years of age.
b. Private elementary schools - the man who had a licence taught in his private home.
c. Schools for the poor— schools for the children of poor people who could not support themselves. The rich contributed money to these schools.
d. Village elementary schools — schools in the remote areas. The teacher omitted a part of curriculum or attended evening school.
e. Girls' elementary schools — schools for girls in which they were taught handicrafts and ordinary subjects.
f. Ordinary elementary schools — It was divided into lower and upper schools.

4) The subjects of elementary schools

At first there were about 20 subjects. They were revised soon and only a few of them remained. I'll introduce something about arithmetic here.

Arithmetic — order and the four operations; they were explained in the western style.

Geometry—the subject for upper schools;

Arithmetic was taught using a western-style method and avoided teaching how to use the abacus.

In the lower elementary school systems, it was prescribed that elementary education was compulsory. Since all the laws had not yet been put in order, I think that the school systems remained just a model which could not be followed as an ideal at that time. Not all the children of school age could go to school.

In 1883 the percentage of school attendance was about 50%. It was still a very low rate at that time.

In 1870 compulsory education started in England for the first time in the world. Two years later the school systems started also in Japan, so it may be interesting to note that compulsory education started in Japan next to England, but elementary school education was already spreading in advanced countries.

B. The Elementary School Syllabus

Following the proclamation of the school system laws, the elementary school syllabus was proclaimed.

It was a little similar to the course of study which was introduced from the USA after World War II. But it was just the syllabus before World War II, or the one which was used in countries except the USA.

The syllabus is as follows:

The elementary school was divided into two stages — an upper and a lower one. The lower one was from the age of 6 to 9. The upper one was from the age of 10 to 12. So the children studied for eight years. The course of the lower elementary school was divided into eight grades. The term of each grade was six months. The contents of each grade were given in this period. But as it was only a model, it was recom mended to revise and to use it in each prefecture.

Next, I shall introduce arithmetic only. I can only find in the eighth grade the same course of study of arithmetic as today's. But I cannot find it in the first grade to the seventh grade.

- the 8th grade: 6 months, 5 hours a day, 30hours a week, except Sunday; Western-style arithmetic: 6 hours a week; Using textbooks, the teacher wrote the Arabic numbers, order and the fundamental calculations of addition and subtraction on the blackboard, and the children wrote them on their paper. Children practised the calculation of figures and mental arithmetic every other day. When a teacher made the children do mental arithmetic, children could not use paper, and only answered questions on the blackboard. The questions which were answered in exercises the day before remained unerased on the blackboard, and the next day, all children were made to do the exercise again.
- the 7th grade: 6 months; arithmetic: 6 hours a week; to teach multiplication and division as in the 8th grade, and to do exercises on calculation of figures and mental arithmetic every other day;
- the 6th grade: 6 months; arithmetic: 6 hours a week; to teach multiplication and division;
- the 5th grade: 6 months; arithmetic: 6 hours a week; to study the application of the four operations, and to do the exercises on the calculation of figures and mental arithmetic every day;
- the 4th grade: 6 months; arithmetic: 6 hours a week; to teach the four operations with compound numbers;
- the 3rd and 2nd grades: 6 months; arithmetic: 6 hours a week; to teach fractions;
- the 1st grade: 6 months; arithmetic: 6 hours a week; to teach fractions and proportions; review of each subject: 2 hours a week; to review all subjects studied before.

After passing the test, children went to upper ele- v o l u - me mentary schools. Children who could not pass the test studied for six months more in the 1st grade.

The lower elementary school was more modern ised compared with the schools of the clans and «Terakoya». But the Ministry of Education did not consider the unification of teaching methods all over Japan soon.

In the syllabus of the lower elementary school, arithmetic consisted of teaching western-style arithmetic and not teaching how to use the abacus. But at that time there were few who could teach westernstyle arithmetic, so the government took the measure of making plans to train teachers. There was a big objection against only teaching western-style arithmetic and not teaching to use the abacus.

There were few books about western-style arithmetic till «Shogaku sanzitsuyo» was published by the Ministry of Education. So it introduced «Hitssan kunmo», «Yozan sogaku» as textbooks, and explained a little about the teaching methods. I do not think all the teachers could teach the children. In the Ministry of Education there occurred discussions about it. In 1874 it gave a notice, «We do not intend to use only western-style arithmetic, but we will let the children study the abacus, too.»

Arithmetic in the syllabus of elementary schools did not limit the size of numbers for each grade. Today it is said that each teacher determines the limit of numbers according to the children's ability. It is not necessary to show the details. At that time, not everyone knew about the modern school, so it could not be helped. It might be unavoidable when the development of children was unknown.

Geometry was the subject for upper elementary schools. Measurement was only thought as the adaptation of calculation. Now I shall introduce to you «Hitssan kunmo» to show how it was taught at that time.

It was published in September 1869. The writer was Meiki Tsukamoto — a geographer. He was a talented man who played an active part in the navy and bureaucracy. He was the first writer who offered arithmetic in a systematic and modernised textbook,» said Kinnosuke Ogura. It was because he studied western-style arithmetic from the beginning — he was not a man of the abacus. This book was published as a primer, but it was of high level.

This book was the first that showed the style of the subjects as it is common now.

- The first stage is a general explanation.
- The second stage is a detailed explanation of the methods with examples.
- The third stage are exercises of calculations.
- The fourth stage are exercises of applications and problems.

The book contained four volumes. For each volume existed another book which contained formulas and answers.

- the first volume: number, four operations; the second volume: fractions;
- the third volume: proportions;

Now I shall introduce to you a part of the first

1) Number

The number in Chinese character

The large number (larger than 10)

The decimal number (smaller than 1)

The cardinal number

The notation

the number placing and notation of the decimal system

2) The four operations

Addition: the method of calculation of figures, the addition of numbers of 4 or 5 figures; the number divides 4 figures; applied problems

Subtraction: the same way as addition

Multiplication: explained as addition; the calculation

Division: with 10 to 12 is added to the fundamental multiplication table; to explain multiplication of the number of I or 2 figures to explain the method of division when the divisor consists of I or 2 figures

C. Upper Elementary School Syllabus

- the 8th grade: *arithmetic:* 6 hours a week
- the 7th grade: *arithmetic:* 6 hours a week
- the 6th grade: *arithmetic:* 6 hours a week
  to teach proportionate distribution
  drawing: 2 hours a week
  to draw point, line and regular polygon
- the 5th grade: *arithmetic:* 6 hours a week
  to teach proportionate distribution
  geometry: 4 hours a week
  to teach regular polygon
  drawing: 2 hours a week
  the same as the 6th grade
- the 4th grade: *arithmetic: 6* hours a week
  to teach proportionate distribution
  geometry: 2 hours a week
  to teach line, angle and triangle
  drawing: 2 hours a week
  to draw plain, straight line and volume
- the 3rd grade: *arithmetic: 6* hours a week
  to teach square and square root
  geometry: 4 hours a week
  to teach circle and polygon
  drawing: 2 hours a week
  to draw plain, straight line and volume without shadow
- the 2nd grade: *arithmetic: 6* hours a week
  to teach the calculation of interest
  geometry: 4 hours a week
  to teach comparison with each figure
  drawing: 3 hours a week
  to draw arc, line and volume
- the 1st grade: *arithmetic: 6* hours a week
  to teach series and logarithm
  geometry: 4 hours a week
  to teach practical use
  drawing: 4 hours a week
  to draw a map and others

«Sokuchi ryaku» was the appointed textbook for geometry. It was written by Tora Uryu in 1872. It was for the measurement of land. So part of the book is used for the text. The contents of it were almost definitions and their explanation.

IV. TeacherTraining

Positive educational policies of the Ministry of Education were to build schools, to arrange syllabuses and to train teachers.

To get rid of defects of traditional education, it was necessary to adopt western-style training and to train teachers by foreign teachers.

In September 1872, the normal school was built in Tokyo and lectures began. The following April, the attached elementary school was established. This was the first attached school for the pupils of the normal school. It was used not only to practice teaching but also to investigate teaching methods. Then it developed into the research center of the normal school and the model school for all prefectures. At that time, the contents of teaching in normal schools were mainly concerning teaching methods. In June 1873, it adopted academic subjects of study. Until then the term of the school year was not determined, so pupils could graduate according to the results at any time. They were dispatched to each prefecture as teachers. The term of the school year for upper and lower schools was determined to be two years and each grade was divided into two stages.

As schools were built in each prefecture, there arose a problem of shortage of teachers.

In general, people thought that the contents of elementary schools were three subjects: reading, writing and abacus just like in «Terakoya» and other private schools. So teachers of punctuation, writing and abacus were employed for special subjects.

At that time it was easy to get teachers from «Terakoya» and other private schools. The Ministry of Education made efforts to train prospective teachers. In 1874 each prefecture established teachers' training schools without permission of the Ministry of Education. About 46 of this kind were built all over the country.

Many books on education were published by the Ministry of Education. It thought that it was of no use only to teach teaching methods, but it was necessary to explain to newly appointed teachers in detail the book of «The Primer of Elementary Schools».

Then the principal of Normal School, Nobuzumi Morokuzu published «The Required Manual for Elementary School Teachers», and many other books of this kind were published.

On the other hand, M. M. Scott taught carefully the teaching method of modernised schools, not through the books. The students who graduated from the schools went to provincial normal schools to teach the teaching methods. But as the percentage of school attendance increased, the number of schools increased. So there was a shortage of teachers for a long time.

It was necessary not only for teachers but also for the people to have knowledge about modernised schools — «What is education? Explanation of the mission of the school, the system and management of modernised school and teaching method in details.» I introduce one of the popular editions of the book, «The Required Manual for Elementary Teachers» written by Nobuzumi Morokuzu in 1873.

In this book he explained the important duties of teachers in three items - «The teachers arrange the seats of pupils according to their results. They move to upper classes after the test. Where does the teacher stand in the classroom? How are the desks arranged? etc.»

This description was easy to understand for teachers. It may have been difficult to teach the style of modernised school lessons and to get rid of the «Terakoya's» teaching concept.

Now I shall introduce a part of arithmetic education.

The 8th grade: arithmetic

To show the picture of numbers; it was necessary first to know how to read the numbers and to teach Arabic numbers. Teachers wrote both numbers on the blackboard and asked the pupils to read them or teachers read them themselves. Then children wrote on their slate board. Afterwards, some of them wrote on the blackboard. Teachers checked them and said to the children, «Raise your right hand if it is right.»

The 6th grade: question and answer

Using pictures about shape, volume, line and angle, teachers taught shapes and surfaces of material or the kinds and names of lines etc.

V. «The Book of Elementary Arithmetic»

The Book of Elementary Arithmetic was chosen as the textbook for the elementary school syllabus of the normal school and the elementary school syllabus of many prefectures. It was written by the Tokyo Normal School and published by the Ministry of Education. It was a progressive syllabus compared with the textbooks of other subjects at that time. It was considered to be one of the best books on arithmetic education in the country.

The textbook was much influenced by W. Colburn — his thought was based on the «intuition» of J. H. Pestalozzi.

In this edition the main editors were M. M. Scott and C. Davis. It was used in many prefectures. It was excellent, but it was doubted whether it was actually used by teachers.

In March 1873 the first volume was published; in April 1873 the second volume was published;in May 1873 the third and fourth volumes were published; in September 1876 the fifth volume was published.

I introduce a part of it.

The first volume

«The Elementary School Syllabus» of the normal school is taught in the 6th grade. In the 7th and 8th grades, the primer of elementary school — numbers, the memorisation of the calculation tables of addition, subtraction and multiplication were already taught.

Japanese numbered

Arabic number

Explained in two pages, the first edition of the book written by W. Colburn was published in 1821; it was used for 60 years with several revised editions. The last version of '84 was published after his death. The Book of Elementary Arithmetic was, although it was published in 1873, similar to the hook of '84. Material selection and explanation of the problems were like that of '84. Considering the fact of M. M. Scott's coming to Japan, it may have been modelled after the book of '63. In the first volume, children study addition - example: add 1 to 1-10 and 2 to 1-10. Question and answer were just as in the book of '63.

So, how did M. M. Scott see the Japanese? Recently, the following letter of M. M. Scott was found at the Griffis collection in Rutgers University. A part of his letter to Griffis is cited below.

«You ask me what I think of the Japanese after thirty-five years' experience of them. I may say that I always had a very high opinion of their intellectual qualities but had some misgiving as to the practical application of what they could so easily learn.

Those misgivings have been completely brushed away from my mind. They have proven themselves great in nearly every department of human effort and I predict for them even in the near future greater achievement still. When I left Japan it was with much regret at what I thought then to be disserverance of a ten years' acquaintance much appreciated by me, but I have had now twenty years' experience with a large number of Japanese in Hawaii, with a different class indeed, but still with their able and amiable official here.

You are quite right in thinking this country a very interesting one. I call it 'a museum of ethnology'. It would pay you to take a vacation and come here in the near future, and with your powers of observation and your slashing pen you could show us to ourselves as others see us. Pray do come. I will give you a welcome.»

VI. The Situation of Arithmetic Education at the Beginning of the School Systems

A superintendent visited schools throughout Japan.

How did he feel?

A. The report of the superintendent D. Murrey

He said, «There remains traditional style of education.» But he thought that it was better to change only slowly. I think he understood that making gradual progress was better for Japan. He held the same opinion concerning the abacus.

B. The situation in the country

Over a short period of time, each prefecture established training centers and normal schools to lessen the shortage of teachers. Each prefecture devised the method of a circuit teaching independent of the school administration.

Developing school systems in the Meiji Era meant fighting against poverty. They were hard pressed to pay money for education, but realised the value of education gradually and became interested in it.

All the heads of the Ministry of Education made a tour of inspection. It was because people complained of the school systems, and also because there arose a tendency of revising the syllabus of schools.

The school buildings were temples or people's houses. The shortage of teachers was a crucial problem. So the excellent children who left elementary schools with an excellent score were adopted as teachers. The children aged more than 15 years were adopted as assistants. The children aged more than 13 years were adopted «leaders in the lesson». Their ability was just the same as the 6th graders of today.

As the subsidy of government and prefecture was very small, the money for school was expanded for the elementary school district. Farmers and fishermen in those days were so poor that they could not afford it.

There was a time when there were no blackboards and notebooks in the classrooms. Children wrote the numbers and words on trays which were filled with sand or rice bran.

Some of the teachers were not competent to handle the four operations with calculation figures and did not understand why the product of the decimal was less than the multiplicand. When a child learned the Arabic numbers, he asked, «Is number 6 just like the shape of the nose?»

There were a few regular teachers who began to study themselves in each prefecture.

C. Promotion and examination

In elementary schools, pupils had to pass the examination to go up to an upper grade. They took the examination for each grade, and one who succeeded in the examination could go up to the higher grade.

When the pupils left the elementary school, the middle school and the university, they had to take the examination to leave school.

Each prefecture established «the test of the elementary school». There were many books on teaching methods which were published by a private company. I found the background and the method of pedagogy in them.

D. Conclusion

The leaders of the Meiji government were overwhelmed with civilisations of foreign countries. They thought that there was not a moment to lose to equalise the standard of education to catch up with the advanced countries.

The government made efforts especially to establish elementary schools compulsorily, and also to let children go to school, but they did not know people's opinion on education.

The government intended to change the existing system to develop modernised schools in Japan.

On the Value of Mathematical Education Retained by Japanese Society as a Whole

Takashi Izushi and Akira Yamashita

We would like to talk about «On the Value of Mathematical Education Retained by Japanese Society as a Whole». This is a research on how school mathematics, which students learned at junior and senior high schools, has been retained by them when they grew up into society. The purpose of the study is to try to get a fundamental viewpoint for the organisation of curriculum for the department of mathematics in the future.

1. The History of Mathematical Education in Japan

We think that the history of mathematical education in Japan may be divided into the following six stages:

The first stage

This stage was the period of about 40 years of the Meiji Period (1868 — 1911) through the Taisho (1912—1925) to the beginning of the Showa Period (from 1926). In this stage, the main objectives in mathematical education were to get skills in calculating, to train thinking power and to gain practical knowledge.

The second stage

This stage was the period of about ten years in which mathematical education was seriously affected by the Perry Movement. The main objectives were to develop the concept of function and to foster one's power of space observation. World War II took place during this stage.

The third stage

This stage was the period of about ten years after World War II and was influenced by the USA. The method in mathematics teaching was focused on daily life experience. In this stage, the six-year term of compulsory education was extended to nine years.

The fourth stage

This stage was the period of about ten years which was a period of review of the former stage. Mathematics teaching was taken as the matter of mathematics systems seriously. And Japan's economic growth began in this stage.

The fifth stage

This stage was the period of about ten years in which mathematical education was seriously affected by modernisation. The percentage of the number of students going on to high school was over 90%.

The sixth stage

We are now in this stage, the period of about ten years which is a period of review of the modernisation efforts. And efforts are made in mathematics curricula to meet the differences among the students.

2. Purposes

The purposes are classified into the following three.

The *first purpose is* to examine the following: Do they remember or understand mathematics contents which were learned at junior and senior high school?

The *second purpose is* to examine the following: What kind of contents of mathematics are useful to their work?

The *third purpose* is to examine what may be called formal discipline. In the first stage of the history of mathematical education in Japan, formal discipline was emphasised and Euclidean geometry took a great part as formal discipline for a great guiding principle.

So, we examined what impression was made on them. From the examinations mentioned above, we try to get the mathematics contents which are retained by Japanese society, and we would like to use their knowledge to organise the curriculum for the department of mathematics.

3. The Way of Examination

The examinations were carried out twice.

(1) The first examination

This examination was carried out in 1955. The time was during the fourth stage of the history of mathematical education in Japan.

a) Participants

The participants are members of society who learned mathematics before the third stage. They have contributed to economic growth in Japan, more or less. The number of participants was 976 and they were sampled from the whole society according to their occupations: technologist, teacher, specialist, administrator, office worker, farmer, fisher, seller, etc.

(b) by Content

They were asked to solve problems connected to the following contents:

1 Calculation (including positive and negative number, literal expression)
2 Round number, percentage
3 Proportion, reciprocal proportion
4 Fundamental figure
5 Solid of revolution
6 Scale
7 Projective figure
8 Word problem (equation of the first degree)
9 Congruence and similarity of triangles
10 Statistics (graph)
11 Pythagoras' theorem
12 Trigonometric function
13 Coordinates
14 Word problem (simultaneous equation)

Furthermore, they were given the following questionnaires corresponding to the problem: «If you understand the content related to this problem, is it useful to your work?»

(2) The second examination

This examination was carried out in 1982. A part of the result of this examination was presented at the

ICMI-JSME Regional Conference in Mathematical Education (1983, Japan).

a) Participants

The participants were graduated from a senior high school belonging to a national university. This senior high school is an eminent school for the graduates going on to college. The 62 persons of the object were chosen corresponding to the years they were in senior high school, 19 persons from the years of the third stage, 18 persons from the years of the fourth stage and 25 persons from the years of the fifth stage. Their occupations were technologist, scholar, doctor, etc. and they contributed to the improvement of technology and science in Japan, more or less.

b) Content

They were asked to solve problems which were mainly related to elementary geometry, because we wanted to make a study of formal discipline.

The problems were connected to the following contents:

I questionnaires of elementary geometry
II 1 perpendicular of line and plane
2 parallel
3 sum of internal angles of triangle
4 projective figure
5 equivalent transformation
6 condition of triangle construction
7 congruence and isosceles triangle
8 measurement
III logic
IV axiomatic method
V non-geometric model of the axioms of the affine geometry

4. The Outline of the Examinations

(I) The result of the first examination

Table 1 : Correct Answer (%)

Occupation	Content													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
a	75	84	84	86	81	68	50	81	78	77	63	56	62	70
b	81	83	65	87	82	61	44	70	74	66	57	48	62	58

a specialist and administrator (218 persons)
b office worker (219 persons)

Table 2 : Answer That is Useful to Your Work (%)

Occupation	Content													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
a	49	55	67	73	51	75	55	67	52	67	54	59	37	60
b	47	58	52	47	38	70	39	57	43	69	53	40	29	56

a specialist and administrator (218 persons)
b office worker (219 persons)

(I) The result of the second examination

Table 3 : Correct Answer (%)

Occupation	Content										
	II								III	IV	V
	1	2	3	4	5	6	7	8			
a	55	93	94	82	86	87	97	88	90	76	64

a specialist and administrator (218 persons)

(3) The outline of consideration

a) Knowledge

Simple mathematics knowledge which was learned in junior and senior high school is well remembered by every person, though many years have passed since they learned it.

In these tables (Table 1, Table 3) only a, b are displaced, but the persons of other occupations answered also more than 40% of the problems correctly.

It is not suprising that a gets higher percentage of correct answers than b. But the difference is not so much.

The problems in which a relatively differs from b in percentage, are connected to proportion and reciprocal proportion. As for the problems of calculation, b is better than a.

The response about the problems of calculation, round number, fundamental figure, and congruence and similarity of triangles, shows high percentages for each person. On the other hand, the problems of projective figures and trigonometric functions, show low percentages. This is a consequence of the times in which they learned. We cannot find any differences between the results of the first examination and that of the second examination.

In the second examination, the objects were chosen concerning the years of graduation from senior high school. We find that knowledge, such as of perpendicular line and plane, which may be observed in daily life are forgotten as the time passes after they learned them.

b) Usefulness

The number of persons who agreed to the questionnaire «If you understand the content related to this problem, is it useful to your work?» is smaller than the number of those who answered the problem correctly .

The contents judged relatively useful for the persons of both a and b are scales and statistics. The contents of proportion, reciprocal proportion and fundamental figure are thought to be more useful by the persons of a.

Many persons answered correctly the problems of calculation and solids of revolution, but few persons thought that these contents were useful. And many persons judged that content of coordinates not useful to their work.

The persons of a more than b think that the contents of proportion, reciprocal proportion and fundamental figure are useful.

c) The way of thinking

It is examined here whether the attitude of deductive thinking which was obtained by learning elementary geometry is still in their mind or not.

IV and V of the second examination are the problems which need deductive thought. V is a problem of a non-geometric model of the axioms of affine geometry.

The result of examinations is satisfactory. It has been a long time since they learned, but they have not forgotten the attitude of thinking deductively.

The result of examinations about equivalent transformation is satisfactory too. The result of examinations about logic is satisfactory.

Y ounger persons replied that they used mathematics knowledge.

But older persons replied that they judged by common sense, though they might use mathematics knowledge unconsciously.

Many members of Japanese society judge that thinking and reasoning powers are developed by learning elementary geometry. And they judge that knowledge of elementary geometry is useful to their daily life, but not to their work.

From these findings, we may conclude that formal discipline is supported by Japanese society as a whole.

What was mentioned above is the results of our examinations, but it can show a viewpoint on what school mathematics should be.

5. Examples of Problems

(I) Problems of First Examination

1. Proportion and reciprocal proportion
   Mark an x if the following statement is false: A train can travel 100 km/h. The distance traveled is proportional to the time traveled. ( )

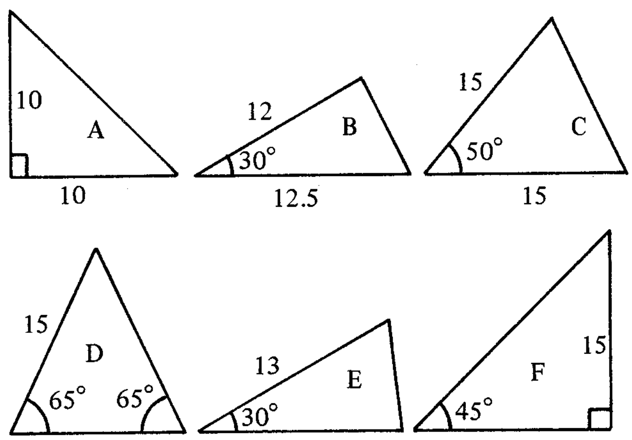
2. Calculation
   a) (+12) - (-7) + (-15)

   b) (0.15 - 3 : 4) -0.3

   c) 4a2b x 2a2b4

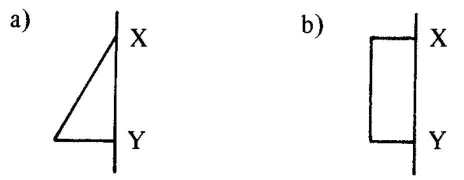
3. Fundamental figures
   Mark an x if the following statement is false: For three lines 1, m, n in a plane, if 1^m and 1^n, then m//n. ( )

4. Congruence and similarity
   Which of the following are congruent or similar triangles?



a) ( ) and ( ) are congruent triangles.
b) ( ) and ( ) are similar triangles.

5. Projective figures
Mark an x on a projective figure which expresses true length.
(a) (b) (c)



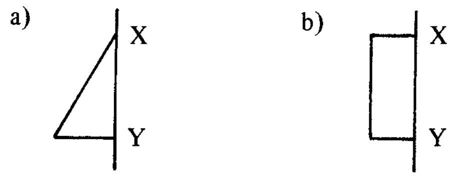
( ) ( ) ( )

6. Scale
Find the actual distance using the scale:1 :50000

a) 3 cm b) 4 cm c) 14 cm

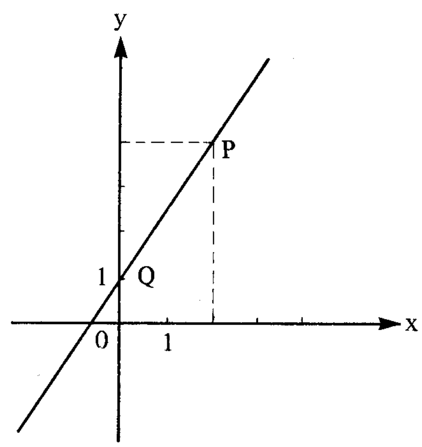
7. Solids of revolution
What are the following solids of revolution?

a) b)



8. Coordinates
a) Give the coordinate of point P.

b) Find the equation of the line PQ.



(2) Problems of Second Examination
Problems of perpendicular line and plane
Describe the definition of the following word.
a) A line is perpendicular to a plane (in a space).
b) A plane is perpendicular to another plane (in a space).

2. Problems which need deductive thought IV Assume the following premise about the organisation of roads and bus stops.
- There are at least two bus stops.
- For every two bus stops, there must be one and only one road which is through them.
- There is one and only one bus stop on the intersection where two roads cross.
- All bus stops don't go along one road.
- For each road and a bus stop (not along this road), there is one and only one other road which doesn't intersect the first road and goes through the bus stop.

According to these assumptions, show in the following order that there are at least four bus stops.
Ql. There are at least two bus stops.
Why? Let these bus stops be named A and B.
Q2. There is only one road which goes through A and B. Why?
Q3. There is at least one bus stop (not along the road in Q2). Why? Let it be named C.
Q4. There is only one road which goes through A and C. Why? Let the road be named b.
There is only one road which goes through B and C. Why? Let the road be named a.
Q5. There is only one other road which goes through A and does not intersect the road a in Q4. Why?
There is only one other road which goes through B and does not intersect the road b in Q4. Why?
Q6. Two roads in Q5 certainly intersect. Why?
Q7. There is only one bus stop at the intersection in Q6. Why?

VA's family go on a journey. Assume the premise about the organisation of cars and pas-
sengers.
- There are at least two cars.
- For every car, there are at least two passengers.
- For any two passengers there is only one car containing both passengers.

For every car show in the following order that
there is a passenger who is not in the car.
1) There is another car which differs from car or. Why? Let the car be named b.
2) A is in car b. Then there is a member other than A who is in b. Let the passenger be named B.
3) Both A and B are not in car a. Why? So there is a passenger B who is not in car a.

In addition to these, the following assumption holds good.
- For each car and a passenger (not in the car), there is one and only one other car containing this passenger but not containing any passenger in the first car.

For each car, there are at least two passengers who are not in this car. Why?
For each passenger, there are at least two cars which do not contain this passenger. Why?

An Evolution Towards Mathematics for All in Upper Secondary Education in Denmark

Ulla Kürstein Jensen

1. Upper Secondary Education in Denmark

A description of the evolution in the teaching of upper secondary mathematics in Denmark would be built on sand without a few introductory remarks about upper secondary education in Denmark.

The traditional Danish upper secondary school called gymnasium is a three-year education based on nine years of compulsory education that contains mathematics throughout. Nevertheless, many students choose to postpone their start in the gymnasium until after the optional tenth year offered by the schools where they receive their compulsory education and originally meant for those who want to increase their qualifications without going to the gymnasium.

The gymnasium consists of two lines, the languages line and the mathematics line, each of which is divided into several branches. Mathematics is compulsory on all branches and is taught at three levels in the gymnasium. The lowest is the one for the languages branches, and there are two for the mathematics branches.

The branching doesn't take place until after the first year, and the maximum number of mathematics lessons per week on a branch is six whereas the minimum is three. The standard timetable on each branch comprises about ten subjects all of which are compulsory.

In 1983 about 20,000 students finished the gymnasium and about 12,000 of these came from the mathematics line. Nearly 4,000 students finished another academically oriented education, the so-called HF-education. Under certain conditions the HF-examination offers the same opportunities for advanced studies as does the «studentereksamen» the final examination of the gymnasium. Mathematics is a compulsory subject in the first year of the HF-education.

In 1983 a total of about 24,000 students, which is approximately 40% of a year of Danish students, completed an upper secondary education with some mathematics.

2. The Evolution that Started in 1961 and a Step Towards an «Upper Secondary Mathematics for All» Course

When describing the evolution in the teaching of upper secondary mathematics it is natural to start in 1961 when new curriculum regulations for the upper secondary school, the gymnasium, were signed. The regulation for mathematics was penetrated by the new-maths-wave and intended for a small proportion of the students, but it was to be applied by a rapidly increasing number of students during the next 20 years. It is important to notice that it restored mathematics as a compulsory subject for the languages line students after a pause of a decade. This turned out to be a small step towards mathematics for all.

The purpose of mathematics for the language students was to give the pupils an impression of mathematical way of thinking and method and to provide them with mathematical tools that could be useful in other subjects at school and during their future activities, so it wasn't only aiming at university studies.

The topics to be taught were: the concept of a function, elementary functions, infinitesimal calculus, computation of compound interest, combinatorics and probability theory.

The first textbooks were very theoretic and the whole course nearly failed completely, but it was rescued by a new textbook at a suitable level.

When the HF-education, that is a type of further education meant as an offer to everybody who wishes to qualify for more advanced theoretic education, came into existence, it was but natural that the purpose of the mathematics regulation 1967 was nearly the same as the one for the languages line students. The course was to be more elementary though, for instance, it shouldn't comprise infinitesimal calculus, but the textbooks included chapters from the textbooks for language students. After a short time, it became evident that fundamental changes were necessary in order to change this compulsory one-year, five lessons per week course to a success.

In the revised mathematics regulation for HF it was mentioned as the first goal to provide the students with mathematical knowledge that could be useful in other subjects and in their daily life, and as the second, to give them an impression of mathematical method and way of thinking. Theoretical algebra and group theory were removed from the curriculum and statistics, probability theory, and binomial test were entered. The so-called free lessons meant for cooperation with other subjects or for elaboration of previously treated material appeared in the curriculum. A textbook for the course was written on the basis of an experiment. This textbook comprised many examples connected to everyday life and it encouraged the pupils to find others. The style of it made it easy to understand also for many who had earlier given up mathematics as too difficult or as uninteresting. It contained few proofs, but it was very good at helping the students to create relevant concept images. The text encouraged the teacher to let the students spend much of the lessons working in groups of three to five persons solving and discussing problems. In this course, mathematics was no longer a terrifying subject.

In retrospect, the new HF-curriculum together with the new textbook and the applied methods of teaching seem to have served as a catalyst for the succeeding evolution. The course was the first course that might be called an upper secondary mathematics course for all.

The HF-mathematics that at the beginning had been heavily influenced by the mathematics curriculum of the languages line now in turn influenced this curriculum little by little. Among the reasons for this evolution should probably be mentioned that the increasing number of students not aiming at university studies or similar advanced studies created difficulties for the mathematics teachers, especially when they were teaching the languages line students, and the fact that teachers often feel more free to experiment when teaching these students than when teaching those on the mathematics line because the former have only an oral exam to pass whereas the latter have to pass an oral as well as a written exam, the latter being centrally set by the Ministry of Education. So the teachers tried consciously or subconsciously to remove some of their difficulties in the mathematics lessons for the languages line students by using ideas or methods from the teaching of the HF-students, and during the last half of the seventies an increasing number of teachers chose an optional mathematics syllabus for their languages line students.

The above mentioned optional curriculum had for some years been used by 90% of the classes when in 1981 it became regulation. Let me state a few remarks in order to characterise it. The objectives are:

The students should acquire:

- some mathematical knowledge which can be of use in other subjects and in their daily life,
- a knowledge of the framing and application of some mathematical models,
- an impression of mathematical methodology and reasoning.

It is noteworthy that it is now explicitly mentioned in the objectives that the students should acquire some mathematical knowledge that can be of use in their daily life and a knowledge of the framing and application of some mathematical models. It is also remarkable that differential calculus in this syllabus may be substituted by another coherent material of the same extent and value if the teacher and the students so wish. Another interesting feature is the so-called free lessons. These approximately 25 lessons can be used for going deeper into the compulsory topics, for working with new topics, for instance some that are connected to other subjects or for providing an introduction to electronic data processing and its role in society. The teacher and the students choose how to use these lessons.

3. Drafts for Courses in Advanced Mathematics that Are at the Same Time Worthwhile Courses for Students Not Intending to Go to University

The evolution on the mathematics line accelerated later. During the last ten years, the number of students choosing this line has increased from about 7,000 to about 12,000, but at the same time the percentage going to university or similar advanced education has dropped considerably. As a result several booklets written by teachers as most textbooks are and offering alternative and often more intuitive and less formal approaches to many topics have appeared during the last five years. The majority is intended to be used either the first year in the gymnasium or at the lower of the two levels. The booklets represent a new interpretation of the regulations. The evolution in HF-mathematics and mathematics on the languages line have inspired the authors of the booklets.

A new interpretation of the old regulation isn't enough to take into account the change in the students' qualifications from their preceding schooling as well as the growing influence of computers and the fact that for the majority the mathematics course in the gymnasium isn't just a kind of an introduction course but the students' final mathematics education. Therefore, the Ministry has just issued a draft for a new curriculum for mathematics for each of the branches of the mathematics line.

The intentions leading to the construction of the draft for the curriculum common to all branches but the one on which mathematics is taught at the highest level, the mathematics-physics branch, were to create a curriculum that:

- within some central mathematical fields shows mathematics as a subject with its own essential values,
- permits an all-round elucidation of the interaction between mathematics and other subjects,
- allows time for absorption in major concepts and correlations,
- allows time to meet special wishes from the class or the school,
- encourages that the content of the lessons should be influenced by the teacher and the class to a greater extent than before,
- guarantees that the student has received an all-round mathematical education with perspective and width,
- prepares the students for a wide range of types of further training for which a foundation in mathematics is required.

Both drafts live up to these intentions by comprising not only a list of topics to be taught but also a list of aspects that are to be brought into the teaching, and various comments on ways and means of teaching. The following objectives, topics, aspects and comments are included in both drafts.

Objectives:
The students should acquire insight into mathematics as a form of cognition and as a means of description.
Topics:
1. Integers, rational and real numbers
2. Plane geometry
3. Functions
4. Infinitesimal calculus
5. Statistics and probability theory.

Aspects:
i) Through suitable examples the students should experience how an algorithmic approach throws new light on the mathematics they work with, and they

should acquire a knowledge of the practical application of electronic data processing.

ii) The students should acquire knowledge of parts of the history of mathematics and of mathematics in cultural, philosophical, and social context.

iii) The students should obtain knowledge of formulation of mathematical models as idealised representations of reality and get an impression of the possible applications of mathematical models and of the limitations in the applications.

iv) The students should learn about mathematics, they should be aware of mathematics as a form of cognition and as a language.

Comments:

As to the comments, I have chosen to include just the following:

- The choice of methods of work is to be adapted to the students as well as the mathematical content, and the students should be acquainted with several methods of work so that they can take part in the choice.
- As to the use of textbooks and texts, it is desirable that the students, apart from reading ordinary textbooks, become acquainted with texts about mathematics. It is also desirable that the students try to read a mathematical text in a foreign language.
- Topics should be approached from different angles. There has got to be deductive sequences as well as intuitive ones. Also, the students should become increasingly familiar with the language of mathematics including symbols and concepts from set theory and logic.
- When planning the lessons, respect should be paid to subjects where mathematics is applied.

The draft for the curriculum for the branch with the highest level of mathematics, the mathematics-physics branch, differs from the previously mentioned by including, for instance, some numerical analysis, induction, mathematics from an algorithmic standpoint and recursion that should provide the students with a general theoretical background for future work demanding and involving the use of computers. It also comprises a considerable amount of free lessons, the content of which has to be chosen by the teacher and the students, and it requires a more elaborate treatment of some topics. Among the additional demands on this branch I should also like to draw attention to the fact that the students should learn to express themselves precisely and clearly orally as well as in writing, and to the fact that the students should acquire an understanding of the deductive nature of the subject by working with proofs in connection with which they should also get the opportunity to construct proofs of their own.

Teachers who want to cooperate and increase experience by testing the drafts, thereby helping to formulate other concepts of them, are going to use them now. Drafts for textbooks are also being published. On the basis of these efforts, curriculum regulations beneficial to many, not only to future mathematicians, should appear in a couple of years.

Fight Against Academic Failure in Mathematics

Josette Adda

In France in recent years, studies on the rate of academic failure have revealed the fact that the education system functions by a process of successive elimination of pupils from the normal streams at each level of orientation.

In the appended diagram, an extract from (12), it can be seen that among children born in 1962 in France there remained at the «theoretically normal level» only 72.2% at age 7, 59.5% at 9, 44.1% at 11, 34% at 13, 21.9% at 15 and 16.1% at 17 (the remainder having repeated years or having been put into marginal-type classes).

Moreover, it appears that, on the one hand, these eliminations concern more selectively children of socio-culturally deprived families and, on the other hand, that the responsibility of «the teaching of mathematics» (and not mathematics themselves) is essential in these orientations, which have the effect of confirming social inequalities.

In order to evaluate this «inequality of opportunity» as far as mathematics are concerned, it has been noted that, for 1976—77 (cf. (2)), 52% of the children of upper executives in the corresponding age group were following the C-stream (i. e. a course with a predominance of mathematics), the rate being 6% for the children of workers; their chances of reaching that particular class being respectively 91% and 23%. Thus, entry to these classes was far from being equal for all and the «socio-cultural handicap» was 2.2 times more disastrous for the C-section than for all the classes.

An examination of the socio-professional category of the family head for students at the Ecole Polytechnique in 1978—79 (cf. (2)) reveals that, out of the 602 students, 422 come from the category of «liberal professions and upper executives», (or 70%), whereas this category represented 8.3% of the French population for the age group under consideration.

As for the final year mathematics specialists of the Ecole Normale Superieure of the same year, one notes that, out of the total of 21 students, 12 had at least one parent who was a teacher.

What a lot of «wasted intelligence» (to use the expression of M. Schiff)!

We shall sum up briefly some research carried out at the University of Paris 7 which aims to analyse *why* certain children fail and others succeed and *how* the process of failure works, so as to find what changes should be made in teaching to remedy the situation.

In order to analyse the phenomenon, it is first of all necessary to be aware that children are not normally in direct contact with mathematics, that be coming familiar with mathematics is achieved by the intermediary of «mathematics teaching», which in fact plays the role of a simple intermediary for only a minority but is rather a filter for the majority. A study of its workings is thus indispensable.

Classes are essentially carried out, of course, not in «mathematical language» but in natural language, and so create numerous external difficulties of linguistic origin (on the semantic level rather than the lexical or syntactic levels as is often thought). These difficulties have been analysed at length by D. Lacombe (cf. his lectures at Paris 7). However, the purely linguistic explanation is still insufficient, and it is also in the pragmatic and the rhetoric of the discourse of the mathematics teacher that we must seek the sources of faulty comprehension and misunderstandings.

First of all, being abstract, the objects of mathematics that are treated, the properties and the relations that are studied can never be seen (in contrast, for example, with the objects studied by the physical and natural sciences) and so the distance between the *signified* and the *signifiers* plays here a role that is more crucial than for any other type of discourse. Signifiers such as mathematical symbols, diagrams, graphic representations are necessary and yet are the source of poor comprehension of the mathematical objects signified (cf. (1), (4), (6), (8)). By studying the «misunderstandings» brought about by this confusion between signifier and signified we have observed the responsibility they bear not only in a very great number of errors but also in the impossibility of acquiring the concepts themselves. For example, for many children (and teachers!) there are no sets without a string and numerous adolescents affirm that 2 is a whole number but neither decimal nor rational, 2.00 is decimal but neither whole nor rational, and 4/2 is rational but neither decimal nor whole!

Another way of representing mathematical concepts is the use by teachers and textbooks of metaphors (cf. (14)) that are supposed to refer to the experience of the student. Almost always far too simplistic really to conform to any recognisable reality, they are nevertheless still too complex not to be burdened likewise with numerous extraneous meanings that block access to mathematics.

Instead of seeking to show through appropriate exercises the abstract and generical nature of mathematical concepts, one type of pedagogy has sought for some years to «make mathematics concrete» through the method of teaching: an absurd enterprise and as such inevitably bound for failure. The initial idea that starting with real-life situations and anathematising them can be a form of motivation in early stages was in itself quite reasonable. The constraints of the school system, however, led to presenting mathematical questions rigged out in «disguises» that were extremely artificial, pseudo-concrete and the source of misunderstandings, becoming thus extramathematical causes of errors in problems claimed to be «mathematical». Even problems of the type «Mummy goes shopping, she buys (...)» are not really natural and the expenditure calculated is often very different from that of actual purchases (unrealistic prices, proportions out of line with commercial usage .. .). Moreover, this variable «Mummy» (each pupil supposedly feeling involved) introduces an emotional factor that is not necessarily positive: for example, when the mother has financial difficulties, has little time to do the shopping, is sick, far away or deceased . . . (cf. (3)). Reactions in the face of these *academic* «situationproblems» are very different from those of students to whom one can give the opportunity to «mathematise» a *real-life* «situation» problem: doing the shopping themselves, for example.

F. Cerquetti has shown that when pupils in an apprentice class for baker-pastrycook have to do all the calculations for purchases necessary for making croissants and for selling them, considerable success may be noted, whereas the same students react against all the artificial «word» problems put to them in textbooks and prefer and succeed better in purely abstract games with numbers (cf. (9)).

Young children have a potential for abstraction which is not exploited. The fact that primary school teachers are often recruited from the students who have the least positive feelings towards mathematics is very worrying in France. It sets up an interlocking process of failure (cf. (8)) and declining performance is one of the most distressing phenomena of our educational system (cf. work in progress by F. Carayol and M. Olvera in particular). For example, the use of clear symbolism is perfectly well allowed as a simplification by young children (cf. the well-known experiments of Davydov in the USSR), but certain ways of introducing badly understood formalism are rejected by students in the secondary system: in fact, when one seeks the causes of rejection in the teaching of mathematics, one almost always finds that it is a question of notions, the presentation of which has been carried out in an inconsistent way, with inner contradictions .

It is important to stress also that class use of questions «disguising» mathematics, a method fraught with errors because of the outside influences that are introduced, is not «socially neutral» and this constitutes a further factor of selection (cf. (5) and (10)). At the beginning of this century, exercises referred above all to a rural universe of landowner adults who exploited their holdings, transmitted inheritances, invested their savings, and so on. Today, there is an attempt to involve the child more and so school exercises refer often to children but these are children living in towns or cities (often the capital), receiving lots of presents, making journeys, and so on. Thus, not only can certain children not be familiar with certain of the elements necessary to understand the questions but, above all, these «disguises» contribute to giving many of them the idea (immigrants or not — some speak of «home-grown foreigners»!) that they are foreigners in this world, this universe of schoolroom problems that they believe to be (the ultimate mistake!) the universe of mathematics. It is striking to see the archetypes that certain pupils propose when they are asked to invent the text of a problem (cf. (3) for example).

Questionings used in *evaluation* do not so much reveal inequality but create it (cf. (7)). The formulation and the presentation of mathematical problems present the same sort of bias as those (denounced for years now) of IQ tests. Moreover, poor results have an all the more disastrous influence in that the present school system sets up a *locked-in process of failure* through the «Pygmalion effect» (self-fulfilling prophecy) and above all by the irreversible streaming off towards poorly thought of types of classes.

The struggle against academic failure in mathematics is not a question of change in curriculum. It requires a concerted attack on the true basic problems, for otherwise all the sources of difficulty can recur, in a more or less serious form, on any chapter of mathematics. The «reform» of recent years has been a good example of this, with the result that all the criticisms expressed at the time of the survey on teaching carried out by the review *L 'Enseignement scientifique* in 1932 are easily transposed into the present situation.

Teachers must become aware of those aspects of their teaching practice which create misunderstandings and lack of comprehension; they must not only have a solid knowledge of the mathematics that they have to teach but also be capable of understanding how these are to be transmitted in the mathematics class and study the ethnological features of that universe where, in the interrelationship between teacher, pupils and mathematics, communication is threatened by interference which can be called «socio-logical» (according to P. Bourdieu).

In order to give all pupils access to mathematical culture with its own specific features, it would be necessary, as for physical culture, to offer to all the pleasure and the opportunity to carry out exercises (here intellectual and abstract ones) (see for example (11)). Above all, let us not forget that we are talking about an integral part of the cultural heritage to be transmitted and that theorems are also works of art: Pythagora's theorem is a classic on the same plane as a play by Shakespeare or a painting by Leonardo da Vinci, and there is in it an aesthetic wealth that we must try to offer to all. Contrary to those who want to confine underprivileged children to «useful mathematics» (in the sense of creating minimal automatic responses with no practice in deductive reasoning), I think that we must attempt to allow all children to exercise a right to abstraction (for which mathematics teaching offers the best opportunity), the formative element in the loftiest activities of the human mind.

Certain mathematicians, conscious of the collective responsibility they bear for the harm done by misuse of their discipline, recognise that it is their duty to react. Academic failure at the present time is no longer solely the failure of pupils, it is the failure of the whole educational system, and, if teachers fail in their struggle against this failure, the responsibility will fall on those whose mission it is to train teachers, in other words, those working in tertiary education. The mathematical community must conscientiously do its duty towards the school system through the initial training of teachers and inservice training (with special emphasis on recent research on mathematics teaching) for practising teachers, as well as by the development and improvement of the forms of support necessary for all those for whom it cannot be provided by the family context.

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1 School Year	2 Total Pop (1000s)	3 Age (yrs)	4 Classes from "CP" to "Terminal": Advance (total)	5 Theoretically Normal Level	Delay 1 yr	2	3	4 and +	6 Total	7 New Classes CPPN	CPA	Short Secondary Cycle: CEP	CAP 3 yrs 1st	2nd	3rd	CAP 2 yrs 1st	2nd	BEP 1st	2nd	8 Total	9 Final Studies Classes (Primary) 1st	2nd	10 Special Educ.	11 Tertiary	12 Total Rate	13 Exit
68–69	832.6	6	6.9	CP 91.3	1.8 (pre-school)				1.8														0.7		100.6	
69–70	835.3	7	7.0	CE1 72.2	19.3				19.3														1.5		100.1	
70–71	838.4	8	7.3	CE2 64.3	22.2	4.2			26.4														2.6		100.5	
71–72	841.1	9	6.7	CM1 59.5	23.5	6.2	1.0		30.7												0.1		3.5		100.4	
72–73	843.1	10	5.4	CM2 55.0	26.8	7.6	1.4	0.3	36.1														4.0		100.6	
73–74	844.9	11	4.8	6th 44.1	34.2	9.5	1.8	0.4	45.9														4.3		99.3	
74–75	846.2	12	4.5	5th 38.8	43.5	5.8	1.4	0.4	51.1												0.1		2.8 1.6		99.3	
75–76	846.0	13	4.1	4th 34.3	45.5	8.6	0.8	0.3	55.2												0.1		1.8 2.6		98.3	1.0
76–77	846.0	14	3.4	3rd 30.6	27.6	8.7	0.5		36.8	10.4	0.8	–	10.7	–	–	–		–		10.9			1.2 3.0		97.1	1.3
77–78	845.3	15	2.6	2nd 21.9	27.1	5.1	0.4		32.6	3.1	6.4	0.8	8.1	8.6	0.1	0.2	–	3.5	0.1	21.6			0.1 2.7		91.2	5.9
78–79	844.8	16	2.5	1st 17.6	15.4	4.8	0.2		20.4	0.4	0.4	0.3	1.4	6.9	7.2	0.7	0.3	9.4	3.1	29.0			0.1 0.9		71.6	19.6
79–80	844.1	17		Term. 16.1	14.0	3.0	0.3		17.3			0.1	0.2	1.4	6.4	0.4	0.7	4.2	7.8	21.2				0.8	55.4	16.2
		18											–													
		19											–													
		20																								

EQUALS: An Inservice Program to Promote the Participation of Underrepresented Students in Mathematics

Sherry Fraser

Mathematics has been called the queen of the sciences. She could also be called the gatekeeper to the job market. Too often, students who might find job satisfaction in a scientific or technical field are unable to enter that field because of inadequate preparation in mathematics. Why is it that students, especially female and minority students, stop taking mathematics in high school, as soon as it becomes optional to do so?

Many women and minority students don't see the relevance of mathematics to their future lives. This perceived lack of usefulness of mathematics contributes to the high dropout rate. If students don't see the need for math they do not take the elective mathematics courses and effectively screen themselves out of many potential careers. Another factor in students dropping mathematics is their lack of confidence in their ability to be successful in doing mathematics. Traditionally, mathematics has been seen as a male domain. In the United States it is socially acceptable, especially for girls, not to be good in math. Unless the student feels competent and confident in doing mathematics she or he will not continue on when the courses become optional.

Students need support from their teachers, counselors, parents, and peers if they are to continue on in their mathematics education. Intervention programs that develop students' awareness of the importance of math to their future work, increase their confidence and competence in doing mathematics, and encourage their persistence in mathematics have the best chance of success. Thus, EQUALS uses these strands - awareness, confidence and competence, and encouragement in its programs.

EQUALS is a mathematics inservice program at the Lawrence Hall of Science, University of California, Berkeley, serving teachers, counselors, administrators, and others concerned with keeping women and minority students in mathematics education. It focuses on methods and materials for the kindergarten through twelfth grade level that will help attract and retain underrepresented students in mathematics. Since 1977, 10,000 educators have participated in EQUALS programs. Six national sites have been created to disseminate the program throughout the country.

The EQUALS program includes multiple approaches to the issues of access and retention. At first, EQUALS mathematics instructors—all former public school teachers—set out to sharpen the participants' awareness that disproportionate numbers of women and minority students decide not to continue on in mathematics in high school and are thus unprepared to enroll in vocational or college programs requiring quantitative skills. To develop a commitment to recruiting and retaining students in mathematics, participants must have an investment in the issues. So we ask participants to investigate specific conditions indicating their schools' performance in bringing women and minorities into mathematics (such as comparing math course enrolments of males and females or surveying students' career aspirations). The participants then become the experts on the situation in their schools. They begin to define the problem and are ready to work on solutions with others.

Secondly, EQUALS provides teachers with learning materials and methods that engage them in doing mathematics with competence and enjoyment. EQUALS participants encounter logic, probability and statistics, estimation, geometry, and nonroutine applications of arithmetic for problem solving. These are areas of mathematics that are relevant for mathbased occupations and in which women and minority students tend to lag behind males and majority students in achievement tests. The learning environment must be one that is cooperative and non-threatening. EQUALS models the kinds of teaching approaches we hope to encourage in the classroom, such as providing time for people to work together on math problems and other new materials; minimizing the fear of failure and encouraging risk taking; providing manipulative materials to use in making abstract math concepts concrete; and helping people develop a range of problem-solving strategies that suit their style of teaching and learning.

Recognizing that a number of people must have a stake in such a program of change, the EQUALS approach has been to build, where possible, coalitions of administrators, teachers, and parents who will work cooperatively to spread EQUALS through their schools. EQUALS participants are strongly urged to teach fellow teachers and parents the math activities they've learned in the program, as well as some of the startling statistics about women's and minorities' disadvantage in employment and earnings. They bring to their classrooms or schools women and minority men who work in math-based professions or skilled trades. These people serve as role models and encourage students to think about their futures in terms of necessary and realistic work.

These activities help teachers convince themselves and co-workers at school of the importance of EQUALS goals and generate support for the often difficult task of inserting EQUALS into a text-and-test dominated math syllabus. The activities also reinforce the idea that EQUALS participants are collaborators in the effort to make mathematics meaningful and productive for students who otherwise may be filtered out of a wide range of occupational choices.

During the year-long program, EQUALS participants keep journals of their experiences using EQUALS in the classroom. The journals reveal that many EQUALS participants identify strongly with the math-avoiding students for whom the program is designed. Again and again the program is experienced as a breakthrough for the teachers themselves. A feeling of personal achievement perhaps contributes as strongly as the practicality of the curriculum and the vitality of the workshops to the program's unusually high evaluations — mean scores of 4.5 and above on a scale of 5 in teachers' ratings of the workshops, and findings that at least 84% of participants apply EQUALS immediately and continually in their classrooms. Schools sending teachers to EQUALS report that in two or three years they observe increased enrollments of previously underrepresented students in advanced mathematics classes and more favorable attitudes about mathematics among all students. Most recent pre- and posttest data indicate that EQUALS teachers and their students are improving in their problem-solving skills as well.

Because of the need expressed by teachers for more experience with computers, and its usefulness as a tool in the mathematics curriculum, EQUALS in Computer Technology was developed and offered for the first time this year. Whether they have participated in a math or computer workshop, EQUALS teachers experience astounding growth, particularly in leadership skills, because they are encouraged to speak up, make presentations, and deliver ideas. Small victories are quickly acknowledged. As the person grows, his or her commitment to the program and the people who fostered that growth remains. As a result, we have advocates everywhere, whose support, in turn, is crucial to us.

What we have learned from the thousands of teachers with whom we have worked is renewed respect for the difficult work they do each day without the public support they so desperately need. Many elementary and secondary schools are alienating environments where teachers are placed in adversary roles to students, parents, and administrators. When they come to a program like EQUALS, which provides a non-threatening, supportive environment where they can take risks, make mistakes, and learn new skills, their response is one of gratitude and enthusiasm. What this tells us is that there is little opportunity for cooperation and creativity in their own schools.

Our task then, as math educators, is to remember and be sensitive to the many difficulties facing teachers as they try to strengthen their mathematics programs and to provide them with the respect and resources they need to accomplish their goals. As teachers grow in their confidence and competence in mathematics, so will their students.

FAMILY MATH

Virginia Thompson

Background

Several years ago, we were asked by teachers in our EQUALS inservice program to think about ways to provide parents with ideas and materials to work with their children in mathematics at home. Many parents had expressed frustration in not knowing enough about their children's math program to help them or in not understanding the mathematics their children were studying. In 1980, the EQUALS program received a three-year grant from the Fund for the Improvement of Postsecondary Education (FIPSE) of the U. S. Department of Education to develop a FAMILY MATH program for parents and children to learn math activities together that would reinforce and complement the school curriculum. Although the activities are appropriate for all students, a major focus has been to ensure that underrepresented students — primarily females and minorities — are helped to increase their enjoyment of mathematics.

The 12- to 16-hour FAMILY MATH courses provide parents and children (kindergarten through 8th grade) opportunities to develop problem-solving skills and build understanding of mathematical concepts with «hands-on» materials. Parents are given overviews of the mathematics topics at their children's grade level and explanations of how these topics relate to each other. Men and women working in math-based occupations meet with the families to talk about how math is used in many occupations; other activities are used to demonstrate the importance of mathematics to future fields of study and work.

Course Content

Materials for each series of FAMILY MATH courses are based on the school mathematics program for those grade levels and reinforce fundamental concepts throughout that curriculum. Topics include: arithmetic, geometry, probability, statistics, measurement, patterns, relations, calculators, computers, and logical thinking. The activities included in this FAMILY MATH sampler illustrate how math topics are approached. A career activity is also included. In any given class, four to six activities are presented for parents and children to do together. They then talk about how they solved the problems and how these activities help with school mathematics. Families are then given these and other activities to continue the help at home. Often, parents will bring in new books and activities they've found to share with other class members.

Participants

Parents learn about FAMILY MATH from their children's teachers or principals, at PTA or school site council meetings, through newspaper articles or church bulletins, or from radio announcements. Classes are offered in the afternoon or evening at schools, churches, community centers, community colleges, or the Lawrence Hall of Science.

Impact of the Program

Evaluation shows that families can and do use FAMILY MATH activities at home and that they have become motivated to continue their exploration of mathematics. Teachers and principals find that FAMILY MATH creates a positive dialogue between home and school and a way to involve parents in their children's education.

The Future

During 1983 — 84, the FAMILY MATH staff will be offering workshops to help parents and teachers to learn how to establish and conduct FAMILY MATH classes; the full curriculum will be published; and a film will be made of the program to help disseminate its philosophy and approach to communities outside of the San Francisco Bay area.

If you would like to be on the FAMILY MATH mailing list to receive notices of available materials and workshops, please send your name and address
to :

Virginia Thompson and Ruth Cossey
FAMILY MATH/EQUALS
Lawrence Hall of Science
University of California
Berkeley, CA 94720

We welcome your comments and suggestions for future FAMILY MATH activities.

Appendix

Growth of FAMILY MATH Classes
in San Francisco Bay Area

Year Offered	No. of Classes	No. of Sites	No. of Families	Total No. of Participants
1981 -82	6	3	46	67
1982-83	11	8	136	197
1983-84 (to date)	16	12	345	654
Total	33	23	527	918

FAMILY MATH Trainer-of-Trainer Workshops
at Lawrence Hall of Science

Year Offered	No. of Participants	% Educators	% Parents (w/o direct educational responsibilities)
1983	115	77%	23%
1984	133	85%	15%
Total	248		

Evaluation of the courses, through observations and a follow-up questionnaire, evidences a high level of math-related activity undertaken by FAMILY MATH participants. Over 90% of the 67 parents who attended classes regularly during the first year have played math games with their children and helped them with their math homework; over 80% have talked to their children's teacher about their mathematics progress. Parents have also taken actions for themselves, including getting a math puzzle or game book (50%); a math refresher book (27%); or taking another math class (18%). These numbers compare favorably with the implementation levels observed during the FAMILY MATH sessions. It appears that the math-related activities that are begun during the class are sustained.

In October 1983 and February 1984, we conducted two 2-day FAMILY MATH training sessions for interested parents and teachers. The response to these workshops was overwhelming: 140 applied to the October session and 164 to the February one. The logistics of handling that many people for two days, 6 hours each day, was formidable. Yet, because we could call on the entire EQUALS staff of 9 mathematics educators, we were able to organize people into groups of 30 and take them through the concepts and activities of the 12-hour course. Evaluations of these training sessions indicate extreme satisfaction and high enthusiasm for the organization and conduct of the program. Further, participants were asked how they would use the training by means of a FAMILY MATH Planning Sheet. From the October session, 51% indicated that they would establish or team teach a FAMILY MATH course this academic year and, according to our information, 22% of them have already done so; an additional 16% have firmly scheduled a class to begin this spring. In the February session, 53% stated they would offer a FAMILY MATH course either over summer 1984 or during the 1984-85 academic year. The majority of trainees who did *not* intend to conduct classes said that they intended to use the materials in their classrooms or at home with their children, at faculty and school board meetings, at church, or at community meetings. Project staff will conduct a follow-up of all trainees in late spring 1984 to determine the extent of these dissemination activities.

Mathematics for All is No Mathematics at All

Jan de Lange Jzn.

Under the influence of Prof. Freudenthal's Institute IOWO (Institute for the Development of Mathematics Education) Mathematics for ally has been a much discussed item in the Netherlands for the last decade. Since 1971 lots of materials were developed for primary education by the Wiskobas department of IOWO and since 1974 Wiskovon developed texts for secondary education.

It is not easy to characterise mathematics the way it was developed during that period, but some of the much-used slogans were:

Mathematics as a human activity - Everyday-life mathematics - Mathematics in the world around you.

During the initial years it was not clear how big the influence of IOWA was, but recent research (1984) carried out by Rob de Jong showed that the influence of the Wiskobas-group is very large as of this moment. As de Jong stated:

«The IOWO-Wiskobas paradigm for math education can be characterised as realistic which means among other things: connection with informal strategies of children, using inspirmg contexts and aiming at the comprehension of fundamental concepts. «

«The results of the research: Wiskobas characteristics have been traced to a large extent and in correspondence with the original intentions in five series of textbooks taking 35% of Dutch primary school, and increasing.»

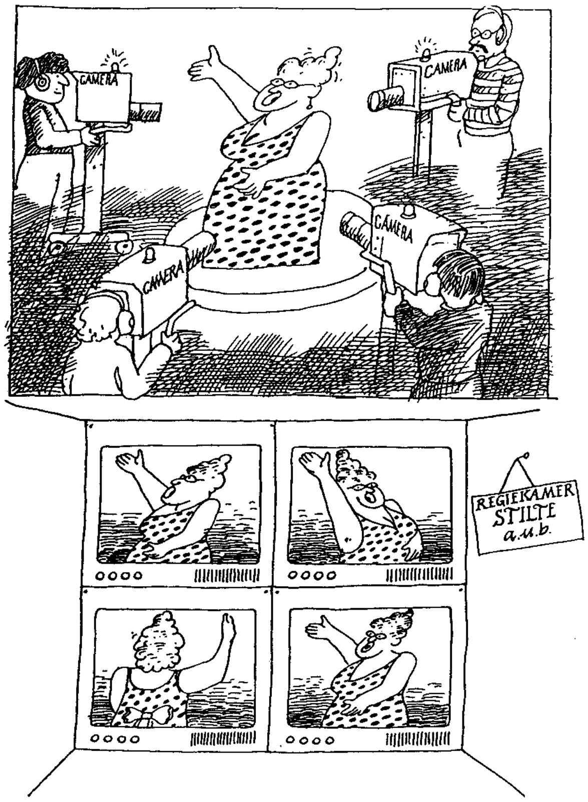
«Moreover: about 80% of the materials used in teacher training can be characterised as IOWO-like.»

«Finally: when teachers are considering a new textbook, 4 out of s take a 'realistic' method.»

This kind of primary mathematics for all may be illustrated by the following examples:

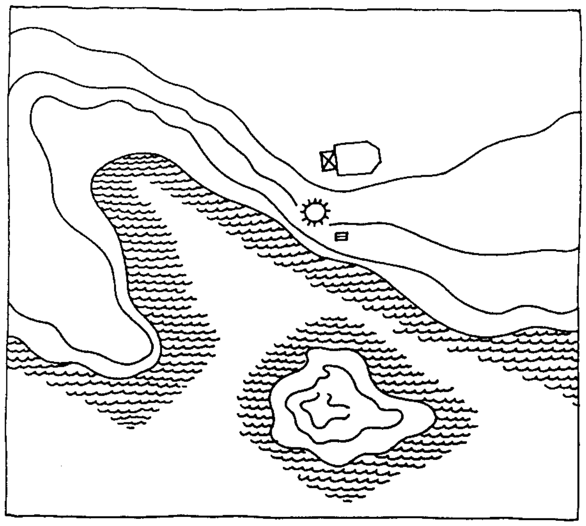
Examples

1. The first example is meant for children of about ten years of age.



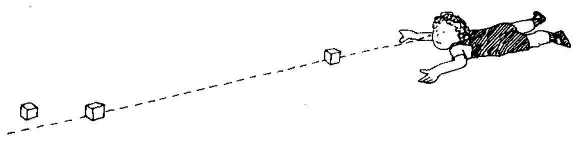
Question: «Which camera presents the picture shown?»

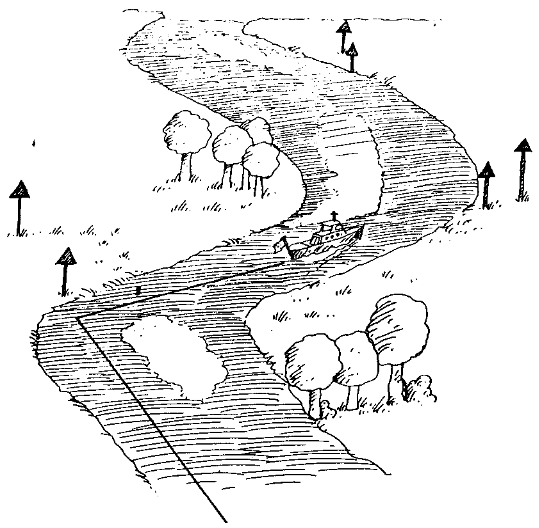
2. Another one, for about the same age-group:
A map of part of the Island of Bermuda is presented:



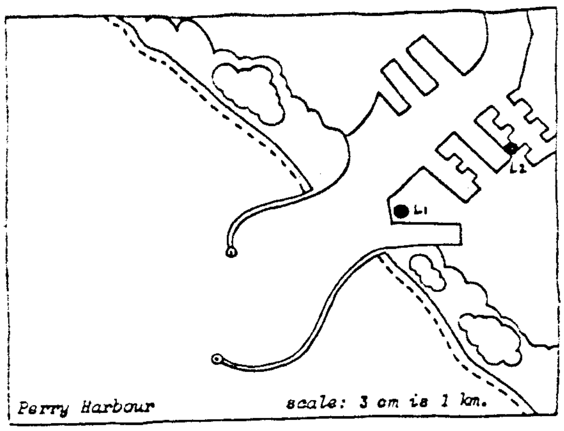
Question: «Which drawing shows the situation as it is seen on the Bermuda?»

3. At a somewhat higher level are the following examples (11— 13 years) about straight lines. A very simple problem: «How can you place three cubes on a straight line» is illustrated in the following way





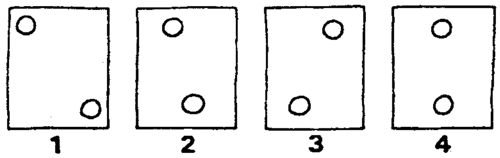
A ship is finding its way on a river with numerous shallow spots. To make navigation easy, a number of signs have been placed on the border. Now you have to sail Jon the straight line» formed by two of these signs that form a pair. As soon as the next two signs are collinear, you change course. This idea is also used when entering harbours.



Here we see a map of a harbour. L_1 and L are lights. L_1 is much higher than L_2.

The tugboat Constance and its tow are reaching the harbour at Perry. The captain of the Constance estimates that they will pull into port in about 15 minutes. He watches the harbour lights very closely, especially L_1 and L_2.

During the last minutes he sees them like this:



Assignment: «Draw the route of the last few miles on the worksheet.»

In a discussion it becomes apparent that the children are capable of understanding the principle. Some of them are even able to place themselves in the position at sea and can translate horizontal information (the L_1, L_2 line) into vertical information and make the right conclusions.

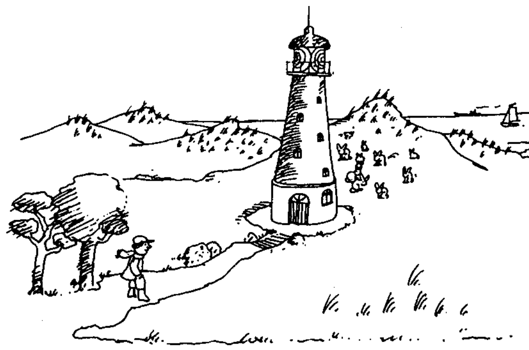
For lower secondary education quite a number of experimental texts were developed by IOWO. Some 20 booklets, mainly of a geometrical background, were the result of five years of experimenting, observing and evaluating. Some of this can be found back in one of the biggest and most influential series of textbooks in the Netherlands. Some continuation of the project — that had to stop when IOWO terminated all activities in 1980 — takes place at the Foundation of Curriculum Development, especially in the field of global graphs.

It is interesting to note that the reactions of teachers to IOWO-like material that is part of an established series of books is at present more favourable than to the original material eight years ago. Via those textbooks they sometimes rediscover the original IOWO-material.

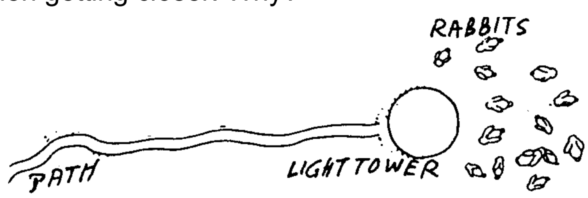
Most reactions are like «Mathematics can be fun» and this seems to surprise teachers even more than students.

Some examples from the original IOWO-Wiskivon materials:

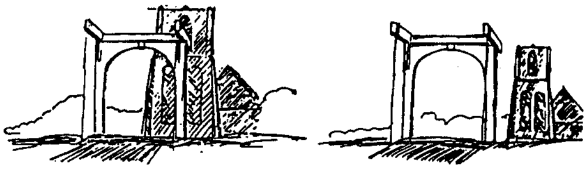
5. The closer you come, the less you can see. That is also the problem of the lighthouse man.



The man walks towards the lighthouse. Behind the tower rabbits are playing in the grass. At home the man tells his children: «When I approach the light-thouse, I get closer to the rabbits. Although they don't run away I see less rabbits when getting closer. Why?»



6. Question: «Is the tower higher than the bridge or not? Explain your answer!» Without proper preparation this is a

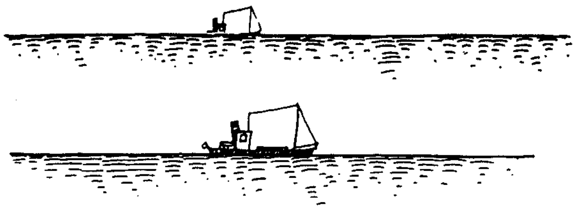


very difficult problem. Everybody knows the phenomenon, but very few people are aware of what really causes it. The designer hoped that a side view would arise more or less

spontaneously, but this was not the fact. But as soon as a side view was suggested the pupils were able to say some sensible things. Although these problems presented to 12- to 13-year-olds still offer many difficulties.

7. Question: «How do you know the earth is a sphere?» Answer: «Because when you are at the beach and a ship is approaching the coast, you first see the upper part, and only later the whole ship.»
Now this answer may not be completely correct, but the next one is quite sophisticated: «When you see a picture of the earth from a satellite, you see a circle.» The teacher: «But then it can be a flat pie?!» «No, because wherever the satellite flies, it always is a circle .»

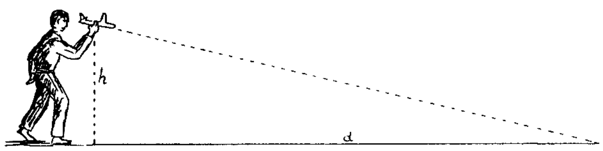
8. Ratio and proportion, as well as the introduction of



angles can be done with shadows, as indicated above. But there are, of course, other possibilities. One of them is experimenting with flying model paper planes.

There are numerous plans to build a successful paper model plane within 15 minutes. This activity alone has, of course, some interesting geometrical aspects. But the planes can be used for further experiments. It is necessary that they fly reasonably, that means more or less in a straight line.

Interesting is to compare this performance of the planes. This can be done by observing how far each plane flies in relation to the height it was thrown from.



It is obvious that for each student the height *h* will be different (and more or less the same for one student) and that the distance flown will vary. Let's compare two planes:

Plane 1 :

h	90	90	90	90	90
d	450	400	360	500	480

Plane 2:

h	120	120	120	120	120
d	600	550	620	550	580

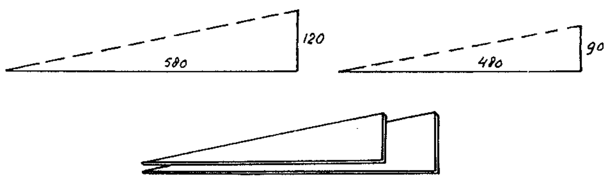
It looks like some more experiments with plane 1 are necessary to make conclusions about *the* distance flown.

Plane 2 behaves very decent. One might say it flies around 580 cm, when launched at 120 cm.

Some additional flights with plane I make it fair to say that plane 1 flies 480 cm when launched at 90cm.

The question arises: «Which plane is the best?»
This leads right into numerous aspects of ratio, proportions, fractions, angles and percentages.

A rather simple way of solving the problem in a geometrical way is by making scale drawings, cutting them out, placing them on to each other and comparing the «glide angles».



The smallest angle gives the best plane! Why?

Finally, some remarks on mathematics for all at upper secondary level. Since 1981 experiments were carried out that will lead to a completely new curriculum for mathematics from August 1985 on. Applications and modelling play a vital role in this curriculum for Math A, aiming at students who will need mathematics as a tool. Math B is for students heading for studies in exact sciences.

Math A seems to be very successful: Applications and useful mathematics starting in reality seem a fruitful approach even for students who are poor at «traditional» mathematics.

More than 90% of the students at present choose mathematics at upper secondary level which is up from 70% in previous years.

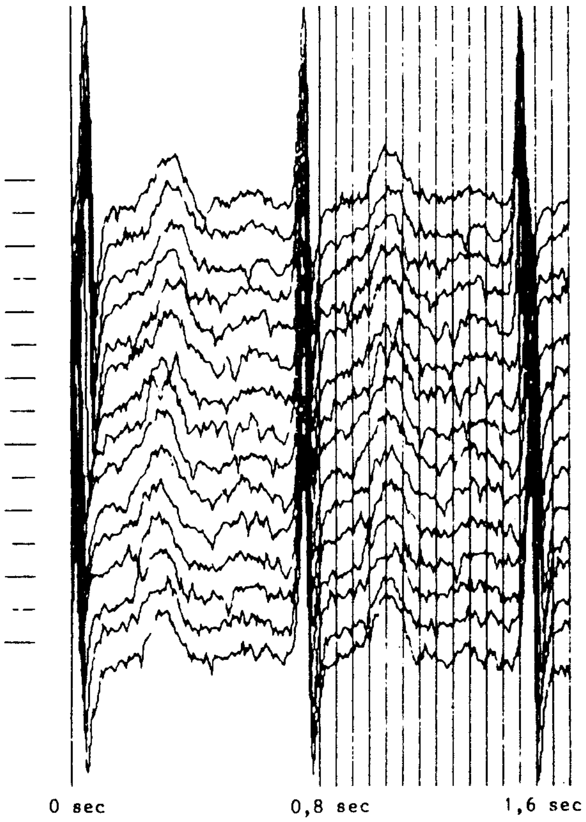
During the experiment 20 booklets were developed for use by the students.

Many of the ideas from these books are to be found in the well-known textbook series in the Netherlands: once more the influence of OW and OC (the research group that can be considered being the successor of IOWO) proved to be considerable.

As a matter of fact, the experiments with Math A were so successful that the vice minister of education considered making math compulsory for upper secondary level.
Some examples of the Math A program are as follows:

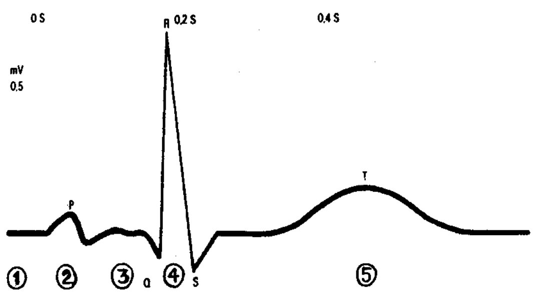
Examples
One of the subjects that is part of the Math A program is *periodic functions.* And as a special case: *goniometric functions.* The latter is not a very popular subject in most courses as we all know. From the experiments we get the impression that embedding goniometric functions in the periodic functions, and in real-life situations, makes the subject much more motivating to the students.

9. The book starts with the electrocardiogram (ECG). An



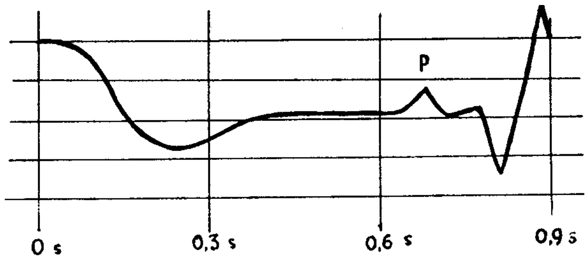
ECG, taken at 16 different spots on the body looks like this:

From this, one can take the average (over the whole body), and finally make a mathematical model that look like this :



In an earlier version, it was not explained what exactly caused the different peaks in the graph. When students asked questions about the heart functioning, (math) teachers were unable to explain. So now we explain the relation between the pumping of the heart and the ECG.

This first periodic phenomenon offers ample possibilities for further questioning. For instance:

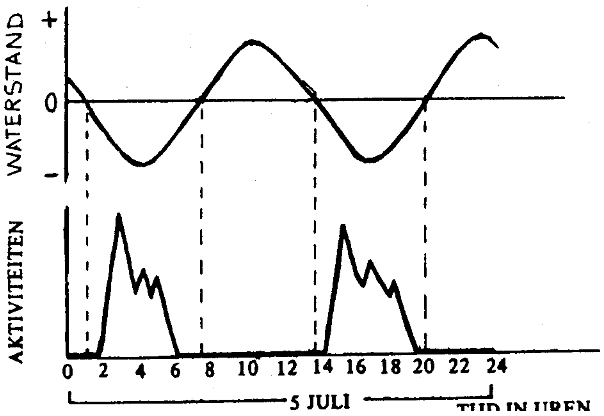


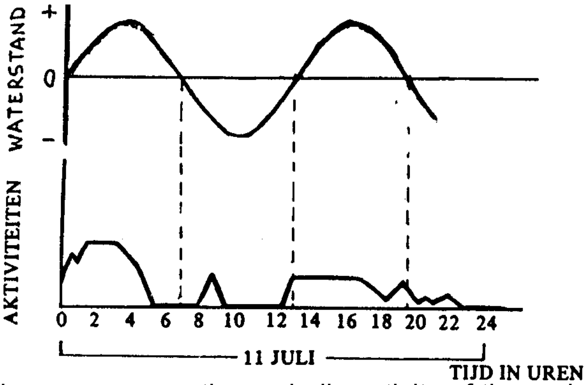
This is a part — one period — of an ECG of someone suffering from a heart attack. The P-top is identified.
Explain in what way this ECG differs from the ECG of a healthy person.

10. An interesting phenomenon that is worth mentioning within the framework of periodic functions is the biological clock.

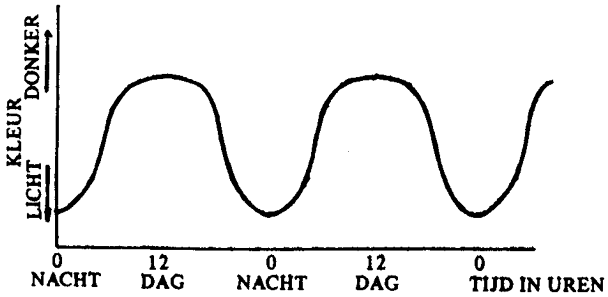
The fiddler crab is very active during low tide, and rests during high tide.

When the crab is taken away from the beach, the graph changes dramatically:



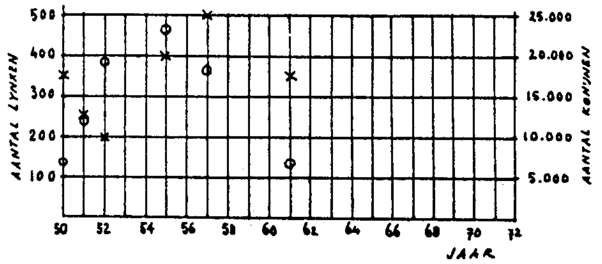


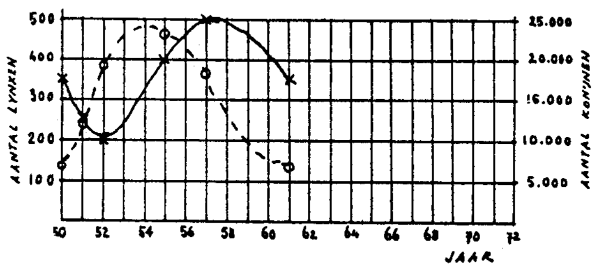
In this case, we say the periodic activity of the crab is caused by an external biological clock.
The same crab has another periodic phenomenon:



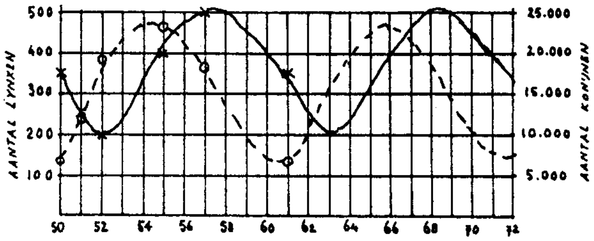
It is coloured darker at daytime and lighter at night. Again, when taken away from its natural environment and placed in a situation with constant light, the periodic colouring remains.
The fiddler crab has an internal biological clock as well.

11. The prey-predator model usually is not found in curricula for secondary education. Certainly not in mathematics. We tried to introduce this model in a very simple way. From a story the students have to draw a graph representing the growth cycle of the predator (lynx) and prey (rabbit) as well.
From:





and finally, because of the periodicity to:



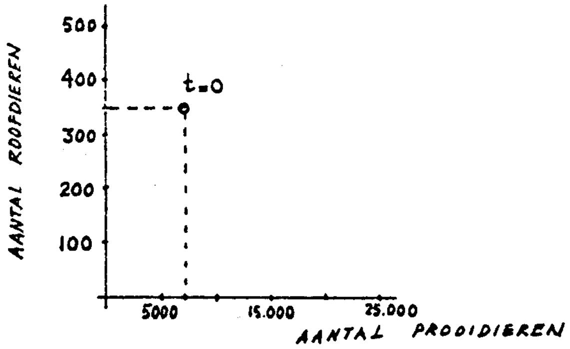
This — not so realistic — story and the graphs are analysed and confronted with real-life graphs, which look pretty much the same and have the same characteristics.

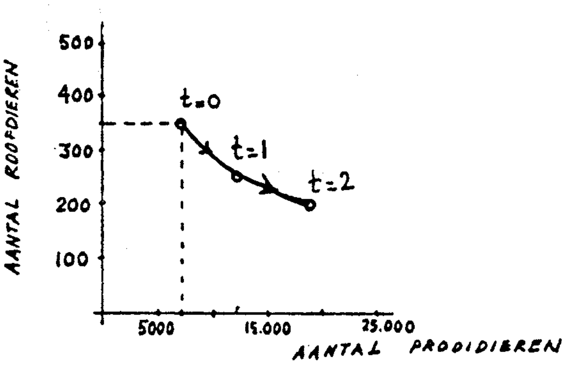
Also questions are posed about a very simple model:

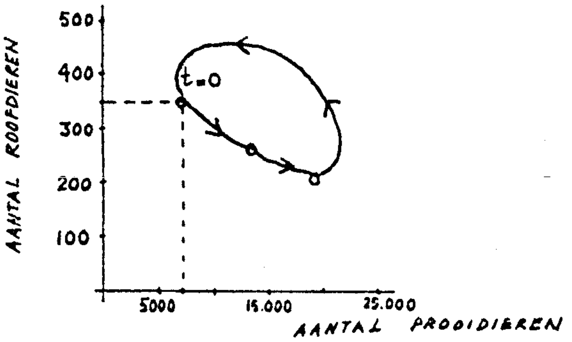
$N_l (t) = 200 \sin t + 400$

$N_2(t) = 300 \sin (t -2 :5p) + 500$

Finally, the children are given another way of drawing graphs of prey-predator models, which emphasises the periodicity of the model:

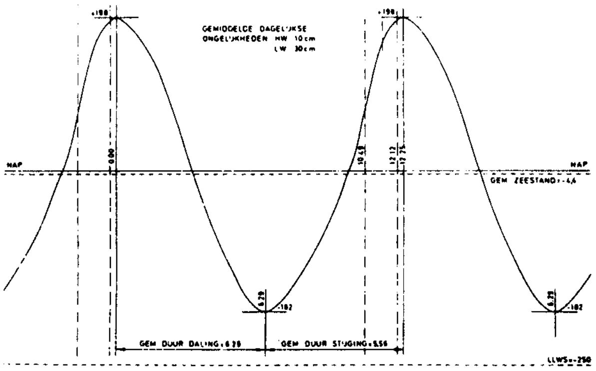






Very important within Math A is the activity mathematising and modelling. This is a complex and difficult matter and offers lots of discussion.

13. The yearly average tide graph of a coastal town in the Netherlands (Vlissingen) is indicated in this graph:



Assignment : «Find a simple (goniometric) model to describe the tidal movement.»
Initially, three rather different models were found by the students (17 years of age):

f(x) = 2 sin 1/2x

g(x) = 190 sin 1/2x + 8
h(x) = 190 sin pi /6.2

Of course, a lively discussion was the result :
f(x), that was clear was a very rough model : the amplitude was «more or less equal to two meters» and the period was 4p or 12.56 which is not «far away» from 12 hours and 25 minutes.
g(x), as the girl explained, was better in respect to the amplitude: The amplitude of 190 cm, together with a vertical translation of 8 cm gave exactly the proper high and low tides which was very relevant to her.
h(x), was more precise about the period. This boy considered the period more relevant «because you have to know when it is high tide». The period proposed by his model was 12 hours and 24 minutes, which really is very close.

After a long discussion it was agreed that:

k(x) = 190 sin pi/6.2 + 8
was a nice model although some students still wanted to make the period more precise.

Many people think that this kind of mathematics is no mathematics at all. When Math A was introduced some teachers asked if they really had to teach this. After working for a year or two with Math A most teachers change their minds: Math A is full of mathematical activities of a very high level. On the other hand, we have to consider that the ultimate mathematics for all is no mathematics — as a separate discipline — at all: It could be possible that especially this kind of mathematics disappears when all other disciplines that use mathematics teach their part of mathematics integrated in the discipline involved.

And so daily-life mathematics for all may disappear in daily-life sciences or general education.

Organising Ideas in the Focus of Mathematics for All

Roland J. K. Stowasser

To Hans Freudenthal on his 80th Birthday

Summary
The history of mathematics offers outstanding examples of simple, and at the same time, powerful *ideas which organise their surroundings,* ideas connected with each other in a transparent network. Rather Pascal's «esprit de finesse» rules the process of thinking, and to some extent, by analogy learning, too.

An impressive example shows that even less able students can take profit by such an organisation of mathematical knowledge. There is more hope that a new quality of mathematics teaching might result from the epistemological and historical point of view rather than from the currently flourishing empirical research, categorising and doctoring merely the symptoms.

About Organising Ideas
There is a lot of lip-service and well-intended general advice for the use of history in math teaching, but very few worked out examples of the kind I would like to talk about.
In the history of mathematics, I was looking out for *ideas*
- influential in the development of mathematics;
- simple and useful, even powerful which at the same time could act as
- «centers of gravity» within the curriculum;
- knots in cognitive networks.

In that sense I call them *organising ideas.*

In the course of history new central ideas developed by reorganisation of the old stocks of knowledge allowing to draw a better general map from those «higher points of view».

«La vue synoptique» brings to light associations hitherto hidden.

This does not work by «longues chaînes de raison» (Descartes). Not Euclid's «l'esprit de géométries» rules the process of cognition and by analogy the process of learning, but rather *a mode of thinking related to Pascal's «esprit de finesse» paving short ways from a few central points to many stations.* *

Pascal himself provides a splendid example. When 16 years old, he reorganised the knowledge about conics handed over by Desargues unveiling the «mysterium hexagrammicum», ever since called Pascal's theorem: the high point surrounded by a lot of close corollaries.

Having cut organising ideas out of the historical context one is left with the hard task to process them into more or less comprehensive teaching modules (or even schoolbook chapters). The products can be problem fields in which very few organising ideas instead of dozens of theorems operate as a means of problem solving.

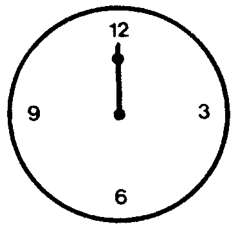
Concentrating on a few simple, and at the same time, powerful ideas which organise their surroundings and which are connected to one another within a simple network, offers help for the less able student, too. His inability derives to a large extent from the fact that he is unable to organise his thought with respect to a complex field in which the connections are presented in the usual plain logical systematical way and where teaching is used to administer only spoonfuls of the subject matter, disconnected and without depth.

An Example on the Idea of Congruence
In German schools, pupils have to learn and apply some special divisibility rules, the end-digit-rules for 5, 25, 2, 4, 8 and the digit-sum-rule for 9 (not more!).

The organising idea behind the different looking types lies hidden away as it was before Pascal's paper about a generalised digit-sum-rule (see appendix to [1]). He wrote the paper in a mathematician's fashion: describing the algorithm by some simple examples (9-, 7-rule) and giving a proof by recursion. His approach is not an appropriate proposal for an interesting lesson on divisibility rules for 11 -year-olds.

Take my approach with the very familiar clock on the face of which, so to say, Pascal's idea, and even the more general idea of congruence comes to light in a very simple way. I quote from [1].

«In the 11-year-olds' daily life experience the clock is just the right thing to start with. I have a big cardboard clock without a minute hand. The hour hand is on the 12.



A pupil is called to the blackboard. He is told to write down the number of hours the hour hand should move. He writes down an unpronounceable number of hours which goes from left to right across the blackboard. Three seconds after the last written figure I put the hour hand on the right hour. For example, the pupil writes:

204501022305312345678902468135790254l403.

I put the hour hand on the 7. The pupils check by dividing through by 12. One page is filled. Plenty of mistakes. In the right hand corner is written the only interesting thing: the remainder.

The pupils know that I am not a magician, especially that I am not good at mental arithmetic. Of course I do not reveal the trick. The pupils will work for it, discover it. A prepared work sheet asking «what time is it», that means asking for

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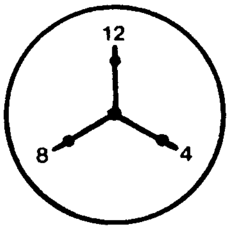
* «(...) il faut tout d'un coup voir la chose d'un seul regard, et non par progrès du raisonnement, au moins jusqu'à un certain degré (...) (Pascal, Pensées, ed. Lafuma 512 : the difference between l'esprit de géométrie and l'esprit de finesse.)

the remainders regarding 12, shows a pattern: the remainders of 12 (Zwölferreste) divided into the powers of 10 (Zehnerpotenzen) from 102 on, are all equal, thank God.
R_{12} (10^n) is constant for n ≥2.

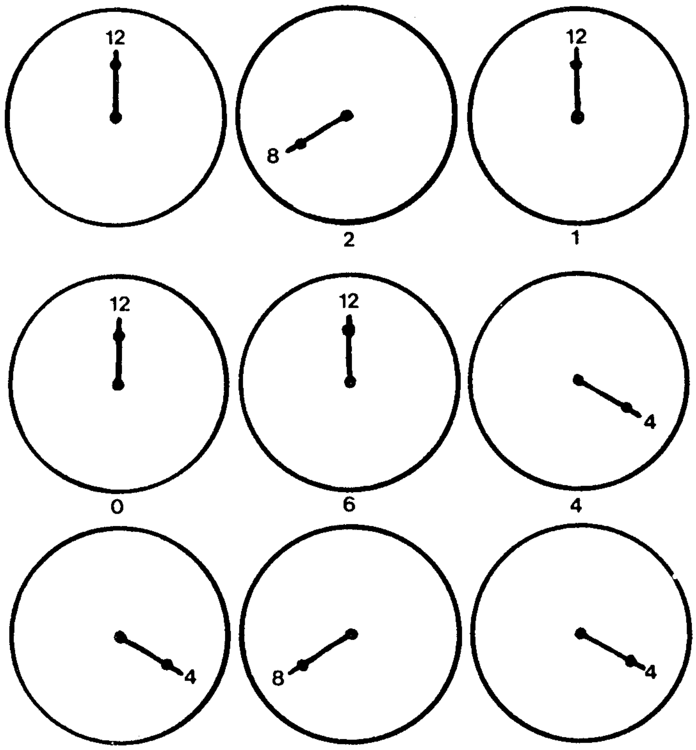
Every 10th power pushes the hour hand 4 hours ahead.

I assume, otherwise it has to be dealt with, the pupils really know what the abacus is, that they can see a decimal number consisting of powers of 10.

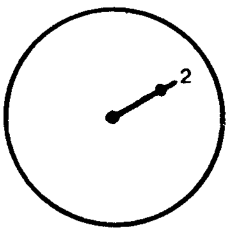
Now my mystery trick in arithmetic is solved. No matter how many digits there are in front, the hour hand simply jumps to and fro among three positions (beyond the tens).



I prefer to do the rotation of the hand mentally instead of on the actual clock. In the example R_{12} (2106437822) my mental arithmetic looks like this:



The 22 hours at the end, being out of the routine, put the hour hand in the final position:



The 11-year-olds even understand my enquiry whether the calculations on the working sheet confirm that R(10000000000) = 4. The reason why - hidden reasoning by induction because of the recursively defined powers of 10 - can be found by 11-year-olds with a little help.
Four hours remain from 10^2 hours after taking away the half days. From 10^3 =10.10^2 hours remain 10 • 4 hours. From 10^3 hours remain therefore again 4 hours after taking away the half days. We proceed in the same way for 10^4 = 10.10^3(...)»

So far, the 11 -year-olds have discovered and fully understood the quick method for remainders by watching the familiar (Babylonian) clock. There will be no real difficulties to transform this method so that it can be applied to arbitrary non-Babylonian divisions of the day (e.g. 11-, 18-, 37-hour-clocks). Pascal's idea has been grasped from the paradigm (12-hour-clock) and transferred, not abstracted from a lot of examples. As a teacher, I couldn't but follow Aristoteles' advice for teaching, as opposed to research, to use but one example, *the* paradigm, which embodies the idea.

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[5] Leibniz, G. W.: Trennlinien and Trennflachen zur Grundlegung der Geometrie. In: Mathematiklehrer 1/1981,pp.28-29. A part of the letter from Leibniz to Huygens (September 8,1679) mentioned in [3].

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The booklet «Recursive Procedures - A ProblemOriented Preparatory Course» concentrates upon the fundamental idea of recursion as a tool for modelling, especially combinatorial situations, rather than for proving.
Difference equations treated before differential equations, linear methods as heuristic means to find say Binet's formula (1843) for the Fibonacci sequence (1202), polynomial interpolation etc. offer a good chance for general practice and preparation of the harder material to follow: in the Analysis, Linear Algebra, Stochastic Courses.
The booklet explains in a manner suitable for l.6- to 17-year-olds a philosophy of math teaching which combines Polya's problem - solving approach and Wagenschein's (Wittenberg's) paradigmatic teaching.

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CSMP: Realization of a Mathematics Program for All

Allan Podbelsek

Introduction

Focus and Direction from Damerow ‘s Paper
In his paper, «Mathematics for All,»* Damerow raises an interesting dichotomy relative to mathematics education — do we continue with a mathematics curriculum designed essentially for a small, elite group or do we develop a program designed to bring more of the essence of mathematics to all students? Stated another way - do we keep the highly selective framework and methods of traditional mathematics education but give up the privileged position of the subject as part of the core of general education; *OR,* do we seek to keep mathematics at the core of the curriculum but find a way of teaching the subject to all students? In spite of 25 years in mathematics education, I still personally believe in attempting to teach as much of the essence of mathematics as possible to the general student population. Therefore, it is issues related to the latter choice that this paper will address.

Basis for this Paper
As a student, mathematics was presented to me in the traditional manner. Because of some effective teachers and my own desire to see things related to one another, I developed an interest in and an appreciation of mathematics as a unified body of knowledge rather than a set of individual, isolated topics. Consequently, in the past 10 years, much energy in my work has been devoted to implement a unified or integrated mathematics program to a subset of students in the school system in which I am employed. It is my hope that someday mathematics programs in the United States will be unified in nature. Perhaps this will happen by the turn of the century along with metrication! Through thinking about mathematics as a unified body of knowledge and dealing with teachers, administrators and community members who were often resistent to such ideas, I developed a strong interest in the very topics that Peter Damerow addresses in his paper.

In the past five years, I have had quite a lot of involvement in the implementation. of an innovative, integrated program at the elementary (K — 6) level. This program is the CSMP (Comprehensive School Mathematics Program). CSMP was developed over a period of several years by mathematicians and mathematics educators from several countries. I believe that one of the motivating factors of its founders was the work of the mathematicians and mathematics educators whose ideas were published in the 1963 «Cambridge Report.» From initial leadership by Burt Kaufman, the federally funded project moved from Southern Illinois University at Carbondale, Illinois to its home at the Central Midwest Regional Educational Laboratory in St. Louis, Missouri (CEMREL).

When Ian Westbury called me last fall to discuss the paper written by Peter Damerow, we discussed the concept of Mathematics for All.» Immediately, CSMP came to my mind as an excellent example of a program designed in the spirit that I thought Peter Damerow had in mind. In a later letter from Peter Damerow I was surprised to learn that he was surprised to know that CSMP was still in existence!

Purpose of this Paper
Because of my belief that CSMP does represent a realization of a mathematics program for all, I proposed this paper. *It is the purpose of this paper to discuss/ evaluate CSMP as a possible realizatiorl of a mathematics program for all.* In order to accomplish this task, it seemed reasonable to enumerate some critical attributes of a program designed around a mathematics-for-all philosophy. In the next section of this paper, I will elaborate on the attributes of such a program based on my study of Peter Damerow’s paper, and my own reading, understanding of mathematics and experiences in mathematics program development.

In the third section of this paper I will describe CSMP and evaluate it with respect to the criteria outlined in the second section. I will also discuss the implementation of CSMP in Louisville, Kentucky, as well as perceived sufficient conditions for successful implementation of this program in general. The third section will be followed with some conclusions and recommendations.

Critical Attributes of a Mathematics-for-All Program

What should a mathematics program for all be like? How should it differ from many of the various, current programs? Is it the topics and content included that must differ or, is it the sequencing of the topics that should change? Is the classroom structure an inhibiting factor in teaching mathematics for all? Is it the background of the personnel asked to teach mathematics that serves as a hindrance to teach mathematics for all? What about the community expectations? What about the mindset of the teachers of mathematics — how they have been trained and what their perceptions of mathematics are?

It would take too much space and time to describe in great detail the attributes of a mathematics program for all. However, I believe that using some broad constructs, it is possible to communicate the essence of what is meant by such a program. Below is a list of possible goals stated for the purpose of analyzing CSMP:

* Paper presented to the ICMI Symposium at the International Congress of Mathematics, Warsaw, August 1983. Zentralblatt fur Didaktik dot Mathematik, 16 (1984) pp. 81-85.

1. Develop the capacity to understand and interpret numerical, spatial, and logical situations which occur in the world in which one lives
2. Develop a scientific, questioning, and analytic attitude toward mathematical problems
3. Develop mathematical knowledge, skills, and understandings relevant to one's personal and vocational needs to include:
   a. problem solving
   b. application of mathematics to everyday situations
   c. estimation and approximation
   d. using mathematics to predict
   e. reading, interpreting, and constructing tables, charts and graphs
   f. computer literacy
   g. understanding and application of basic operations
4. Develop an awareness and appreciation of what is mathematics by recognising and using the following features of the subject:
   a. content dealt with in mathematics
   b. types of thinking used within the discipline
   c. methods of proof
   d. orderliness of mathematics
   e. beauty of patterns and structures of mathematics
   f. power of mathematical processes, patterns, and structures
   g. interaction of mathematics with other areas of human activity
   h. every spiraling development of mathematics through the history of pcople
   i. balance between inductive and deductive reasoning

Most of the above stated goals are self-explanatory. For any program which proposes to teach mathematics for all, it is important to examine how the program would teacher present each of the goals or Subgoals included above.

Evaluation of CSMP as a Mathematics-for-All Program

Brief Description of CSMP

In its recently published report on CSMP, entitled «Conclusions and Recommendations of the Evaluation Review Panel,» the Evaluation Review Panel be gins with a rather concise statement of what CSMP is all about.

The Comprehensive School Mathematics Program (CSMP) is a dramatic curricular innovation in elementary school mathematics. During its development, conscious decisions were made about how mathematics should be taught. The most important of these were the following:

- Mathematically important ideas should be introduced to children early and often, in ways that are appropriate to their interests and level of sophistication. The concepts (but not the terminology) of set, relation and function should have pre-eminent place in the curriculum. Certain content areas, such as probability, combinatorics, and geometry should be introduced into the curriculum in a practical, integrated manner.
- The development of rich problem-solving activities should have a prominent place in the curriculum. These activities should generate topics, guide the sequencing of content, and provide the vehicle for the development of computation skills.
- The curriculum should be organized into a spiral form which would combine brief exposures to a topic (separated by several days before the topic appears again) with a thorough integration of topics from day to day.
- Whole group lessons should occupy a larger and more important role in mathematics class and teachers should be provided with highly detailed lesson plans which lay out both the content and pedagogical development of lessons. Furthermore, training in both the content and pedagogy of the program should be made available to the teachers.

These beliefs about the teaching of mathematics were translated with remarkable integrity into the eventual curriculum materials. CSMP is a model of one very distinctive way of teaching mathematics and is one of the few that can be studied in detail by mathematics education researchers and teachers. Its implementation and evaluation in schools is, in a sense, an experimental test of these distinctive features.

In this K— 6 program, the objects of mathematical study are: numbers of various kinds, operations on and relationships between numbers, geometrical figures and their properties, relations and functions, and operations on functions. Growth in the ability to reason is seen to play an important role in the study of mathematics. CSMP developers argue that in mathematics the development of the art of reasoning goes hand in hand with the growth of imagination, ingenuity and intuition. In this perspective, elementary arithmetic takes on the form of «adventures in the world of numbers. « Individual numbers can assume a personality of their own in this world. Teaching arithmetic shifts its emphasis from an obstacle race for mastery of basic skills to the stimulation of exploring the world of numbers. Skills such as balancing a checkbook have their place in real life but are of little interest to a fifth or sixth grader and are not realistic activities for goals of an elementary school mathematics program. More genuinely rewarding are the stretching of the powers of imagination and the challenging of the mind.

CSMP believes that its program requires a special pedagogy which they call «a pedagogy of situations.» A pedagogy of situations is described as Zone which is based on the belief that learning occurs in reaction to the experience of confronting a situation (real, simulated, or imagined) that is rich in consequences, is worthy of confrontation, and has genuine intellectual «content.» As such, a pedagogy of situations has the following properties:

1. involves children at a personal level,
2. presents an intellectual challenge that is accessible to a broad range of abilities,
3. provides opportunities for creativity,
4. supports experiences that have mathematical content.

The CSMP curriculum presents a large number of varied, yet interrelated situations that provide the experiences out of which mathematics grows. The philosophy of the program is based on the idea that there is no reason why very young children should not have the pleasure of mathematical thinking at an elementary level and of exploring mathematical ideas.

CSMP uses three languages to express mathematical ideas at a young child's level. These languages are: (1) the language of strings, (2) the language of arrows, and (3) the language of the Papy Minicomputer. The language of strings is used to help children think about classification of objects. Its structure is much like that of the Venn diagram. The language of arrows helps the children think and describe relations between objects. Its structure underlies the concepts of function and vector to be studied at a later stage in the mathematical development of the child. The third language is based on a simple abacus and is called the Papy Minicomputer. With this concrete model, children explore and learn number concepts such as place value and develop computation algorithms.

Four major content strands comprise the CSMP curriculum. These are:

The world of numbers

The languages of strings and arrows

Geometry and measurement

Probability and statistics

Attached in the Appendix is a «Summary of the Mathematical Content of the K— 6 Curriculum.»

Some Conclusions of the Evaluation Panel

Below is a summary of the comments made by the evaluation panel.

1. The most important conclusion about CSMP is that it does teach problem-solving skills better than the standard textbook curricula.
2. The original CSMP belief that merely doing computations as part of the problem activities will develop computational skills as well as the traditional program does is not justified by test data. (Modest supplementation removes differences.)
3. CSMP belief that emphasizing problems in a group setting and posing problems directly in the CSMPlanguages will develop adequate skills in word problems is justified by test data.
4. CSMP student effects should be appreciatively larger when more experienced teachers use the revised program. (It was found that often the teacher did not receive sufficient training.)
5. CSMP students probably know more mathematics than the evaluation results indicate. (Tests given do not measure all of the mathematics learned.)
6. CSMP has positive effects on students at all ability levels. (This is important in a mathematics-for-all situation.)
7. The spiral feature of CSMP may be one of the most widely applicable of all features of the program. (More research is needed to determine how the mechanics of the spiral curriculum affect student learning at different points in time.)

To embark upon the implementation of a program as innovative as CSMP is a complex and difficult task. In the United States, conditions are not usually conducive to easily making the kind of changes in teaching mathematics required by CSMP. The Evaluation Review Panel for CSMP summarizes these conditions succinctly as follows:

The status quo of mathematics education makes curricular innovation almost impossible. Content and sequencing of topics have always been heavily influenced by the very traditional, computationally oriented view of mathematics held by many school administrators, principals, and teachers. Recent increased use of commercial standardized tests, and state and locally mandated competency tests, together with public dissemination of the results of these tests, have narrowed the traditional focus further so that, to a large extent, these tests effectively control the curriculum. (...)

This accountability movement has placed increased pressure on teachers to have students achieve these goals, even to the exclusion of other less well measured goals such as problem solving, or less well understood content such as probability. In the future, successful curricular innovations are likely to be limited to those which can provide advance proof of those positive student effects which are valued by the public as represented by school boards and administrators.

Analysis of CSMP Using Criteria for Mathematics-for-All Program

In the second portion of this paper (page 4), several goals that attempt to characterize the essence of the content of a mathematics program-for-all were listed. In this section, CSMP is analyzed relative to these goals. Each of the goals is restated below with comments relative to the CSMP status with respect to the stated goal.

1. Develop the capacity to *understand* and *interpret* numerical, spatial, and logical situations which occur in the world in which one lives.

CSMP: CSMP is rich in situations where students must develop this capacity. However, the program's authors are not so concerned that these situations be in the «real» world from the adult point of view. Their goal is to create situations for this goal which are in the «real» world of the learner which depends on the age and experiences of the learner. CSMP does this very well.

2. Develop a scientific, questioning, and analytic attitude towards mathematical problems.

CSMP: The variety of pedagogical situations through which the content of the program is developed does an excellent job of focusing on this goal. The detailed dialogue provided for the teacher often follows a discovery approach in which the students are pushed or guided to question, analyze, predict or conjecture.

3. Develop mathematical knowledge, skills, and understandings relevant to one's personal and vocational needs includmg:

- problem solving,
- applications to everyday situations,
- estimation and approximation,
- using mathematics to predict,
- reading, interpreting, and constructing tables and graphs,
- computer literacy,
- understanding and applying basic operations.

CSMP: CSMP emphasizes problem solving, estimation and approximation, using mathematics to predict, charts and graphs, and understanding and application of basic operations. Again it takes a different point of view relative to applications to everyday situations because everyday situations are seen from the perspective of the learner. Computer literacy is not included in the written program. Emphasis on understanding the basic operations is strong. Less emphasis on mastery of pencil/paper algorithms is predominant in the philosophy of the program. It is worth noting, however, that great stress is placed on mental arithmetic.

4. Develop an awareness and appreciation of what is mathematics by recognizing and using the following features of the subject:

- content dealt with in mathematics,
- types of thinking used within the discipline,
- methods of proof,
- orderliness of mathematics,
- beauty of patterns and structure of mathematics,
- power of mathematical processes, patterns, and structures,
- interaction of mathematics with other areas of human activity,
- spiraling development of mathematics through history,
- balance between inductive and deductive reasoning.

CSMP: CSMP presents to the learner a broad picture of what is mathematical content. Through teacher led discussions, student activities and written work, the types of thinking associated with mathematics is illustrated and practiced. Through string activities intuitive arguments are practised. However, formal proof is not dealt with in this program which terminates at the end of grade six. The stage is set for pursuit of proof in grades seven and above. Through explorations and problem-solving situations, the orderliness of mathematics is frequently acted out. In CSMPproblem-solving situations, patterns and structures are prominent. Students are frequently asked to utilize a pattern in order to determine a function rule. In addition, the power of mathematics is experienced frequently as students use various structures and models to understand relationships and solve problems. Interaction of mathematics with other areas of human activity is seen in some of the special workbooks. Again, these activities are often those more realistic to the world of the child. The program is spiraling but no emphasis is made relative to history of people. There is a nice balance between intuitive/inductive reasoning and checking guesses in a semi-formal manner.

As is evident in the analysis, CSMP fares very well in terms of the goals stated in a mathematics-for-all program. The greatest lack seems to be in the areas of computation (particularly from a drill and practice/mastery viewpoint), computer literacy, and history of mathematics.

Implementation of CSMP

In the past six or eight years, CSMP has had fairly extensive implementation in the United States with considerable success. To successfully implement CSMP at the local level requires a lot of coordination and special attention to several areas. These are outlined and discussed below.

1. *Teacher Training.* CSMP indicates that 30 hours of training is recommended to help teachers learn about the program and how to teach it Much time is required to bring teachers to understand and appreciate the philosophy of CSMP.
2. *Materials.* CSMP materials are mostly consumable; therefore, there is considerable recurring cost which makes the program more costly to maintain than a traditional one.
3. *Community Awareness.* Because their children will he bringing home materials so different from what is brought home in the traditional programs, there must be a well-articulated plan to acquaint the parents with the program.
4. *Administrative Stagy Awareness*. Local school principals and central office staff must know and understand some of the aspects of this program because they are in positions where they often must explain a program to community members.

In my own school system, Jefferson County Public Schools, Louisville, Kentucky, initial implementation began in 1979 for second and third year students. The initial implementation at the sixth year was in 1982. Overall, the teachers and parents are quite pleased with the program. As was expected, teachers were concerned about students developing a mastery of certain pencil/paper algorithms especially those that are tested on the achievement tests. It is very important in our community to show improvement on achievement test scores and a great deal of the school system's image in the community is determined by performance on these tests. Therefore, some instruction related to computation was provided to students from traditional texts.

The school system's elementary specialist received over 30 hours of training in St. Louis before assuming responsibility for training teachers for grades one through five. In 1982, I spent three days in St. Louis preparing to train the sixth grade teachers.

Early months of implementation for teachers of grades one through five were hampered by the fact that the teachers had at most two days of training. A few teachers received no training. Getting them out of the traditional textbook was not an easy task. Because of these problems, special care was taken to provide sixth grade teachers with more training. Thus, I provided 60 hours of intensive training to nearly all sixth grade teachers involved in teaching the program. These teachers were much more confident in their first year of implementation than were the elementary teachers.

It was very helpful to have an expert from CSMP come to our system to provide information to a large group of community members. Generally, as parents learn and see what the program can do, they are very supportive .

Getting and maintaining materials and supplies has been a challenge in our system. The annual cost of consumable materials threatens the continued use of the program in our school system.

Recently, a traditional textbook which utilizes some of the philosophy of CSMP was chosen as a supplement to the CSMP materials. Perhaps in another five or ten years, other traditional texts will incorporate some of the CSMP philosophy.

Sufficient Conditions for Implementation of CSMP

Based on my experiences over the past eight years, the four areas cited above under Implementation of CSMP» are absolutely essential. The Evaluation Review Panel for CSMP strongly supports this claim. In addition, they indicate that the role of the local coordinator in implementing and managing the program in school districts is vital to the success of CSMP. Without a skilled and influential person at the helm, a solid implementation of CSMP is almost impossible. Someone has to interpret the program and help teachers, administrators, and parents realize the need for and importance of the program.

Summary

Some Conclusions

The purpose of this paper was to provide evidence that CSMP is a realization of a mathematics program for all based on implications of the Damerow paper. According to the criteria developed in this paper to characterise a mathematics program for all, CSMP rates very high. The program is effective with all ability groups and it does much to foster problem solving through mathematizations.

Shortcomings seem to be in the areas of computer literacy and history. In addition, CSMP places less emphasis on computation with paper and pencil than is required by school systems in order for students to perform well on the achievement tests.

On the other hand, CSMP is compatible with some recent trends in mathematics and mathematics education. Some of these trends are listed below:

1. increased emphasis on problem solving,
2. increased mathematics requirements for highschool graduation,

3. need to provide teachers with more mathematics training,
4. increased use of computers in schools,
5. increased interest in discrete mathematics and algorithmic thinking in mathematics.

Some Recommendations

CSMP has much to offer as a mathematics program for all. However, it must be scrutinized and updated in several ways. Suggestions of areas in which change should be considered are listed below.

1. The use of computers must be brought into the program
2. Logo should be used in the text materials
3. More use of history of mathematics should be included
4. Programs should be developed at higher grade levels to sense as an extension to the present program which terminates at the end of sixth grade
5. Ways should be explored to make the program more cost effective
6. The ICME 5 Theme Group on Mathematics for All should explore ways that CSMP and other similarIV. Geometry programs can be supported A. Networks
7. Research should be planned to help develop CSMP. As this theme group considers issues related to the transformation of mathematics education from the training of experts into an essential part of general education, I hope that the contributions made by CSMP toward this end will be examined and valued. I believe CSMP is one of a few, if not the only, ex-emplification of a mathematics program which is built Kin the spirit» of what should be a mathematics program for all.

Appendix

A Summary of the Mathematical Content in CSMP

Kindergarten

I. The World of Numbers

A. Counting

1. Count dots in pictures.
2. Draw a given number of dots.
3. Find the dot picture that corresponds to a given numeral.
4. Play counting games.

B. Numeration

Recognize and write numerals for whole numbers.

C. Order

1. Play a game in which whole numbers are located on the number line.
2. Compare sets to determine which has more elements.

D. Addition and Subtraction

Interpret and draw dot pictures for simple addition and subtraction stories.

II. Probability, Statistics, and Graphing Data

Collect and record data in bar graphs.

III. Problem Solving and Logical Thinking

A. Reasoning

1. Use clues to identify unknown numbers.
2. Recognize and represent the intersection of two sets.

B. Relations and Functions

Interpret and draw arrow pictures for simple relationships.

C. Sorting and Classifying

1. Sort and classify objects such as attribute blocks and centimeter rods.
2. Place dots for objects in Venn diagrams.

D. Patterns

Determine a rule for a sequence of objects.

IV. Geometry

A. Networks

Follow one-way roads from one point to another.

B. Taxi-Geometry

Draw taxi-paths from one point to another.

C. Measurement

1. Compare lengths of centimeter rods and lengths of paths on a grid.
2. Use attribute blocks to compare areas.

D. Transformational Concepts

Work with mirror and «cut-out» activities that involve reflective symmetry.

E. Euclidean Concepts

1. Recognize circles, triangles, squares, and rectangles.
2. Do activities that involve spatial relationships and perspective.

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Mathematics for Translators Specialized in Scientific Texts — On the Teaching of Mathematics to Non-Mathematicians

Manfred Klika

> Mathematics is the unique art and science that enables us to cope with the complexity of economic, social and technical problems in a rational, quantitative way. The education and training of students in this field is an international concern. (Proceedings of ACME 4)
>
> Comprehending that which can be comprehended is a basic human right. (M. Wagenschein)

Introduction

Perhaps you are surprised that I want to present a contribution on a topic which seems to be very specific. Yet in fact I deal directly with the main questions presented in the paper of the Organizing Committee of this theme group [5]:

* What kind of mathematics curriculum is adequate to the needs of the majority, what modifications to the curriculum are needed for special groups of learners?

In this paper, the extent to which experience gained in our project could be transferred to the teaching of mathematics to the majority will be discussed. Starting with specific objectives, I will develop my argument to reflect on mathematical education in the future.

The Structure of the Course «Fachübersetzen «

Because of a large amount of international co-operation, the need for high quality translations has increased in recent years, especially in the physical sciences and engineering. A trend-setting degree program has been set up at the Hochschule Hildesheim for training «Fachübersetzer»—technical translators in specific technical fields (at present limited to mechanical and electrical engineering). The program is aimed at providing the students with both a practically oriented and a theoretically based foundation in linguistic attainments closely connected with knowledge in technical fields.

The students should acquire the capability of seeing specific interrelations within their future fields of activity, of working independently, of working with problems, between disciplines, with a scientific approach. The close integration of linguistic and technological studies is achieved by using the subjects covered in the technical courses of one year as the basis for exercises in translating technical texts in the following year of the program (see the table «Structure of Coursework 'Fachubersetzen'»).

In order for the students to gain the solid foundation necessary for studying technical fields, compulsory lectures are offered on the «fundamentals of technology» during their first two semesters in which mathematics plays an important role.

The program is addressed to students who have a special inclination and ability for languages and who are also interested in physical sciences and engineering. Employment openings for the graduates include: translators in trade and industry, national and international agencies, and publishing companies, and linguists in related fields like terminology, lexicography and documentation.

But there may be a problem in the future here. Far-reaching changes are now also occurring in the field of linguistics as a result of advances in electronics; computers are being used more and more in the work of translation. Already, future technical translators are being introduced to the methods and problems of mechanical translation during their course of study. In the next few years, this component of their study will be undertaken in conjunction with a study of computer science. Both «Fachübersetzen» and computer science could meaningfully work together in linguistic data processing.

Up to now research has provided some impressive results, in particular in the translation of very specialized, greatly standardized texts. Computers will be able to relieve translators to an increasing extent of routine work, so that they will be able to devote all of their time to more demanding linguistic tasks. The linguistic and technical demands on translators will therefore certainly increase in the next few years. But I venture to make the following prediction: computers will not replace translators.

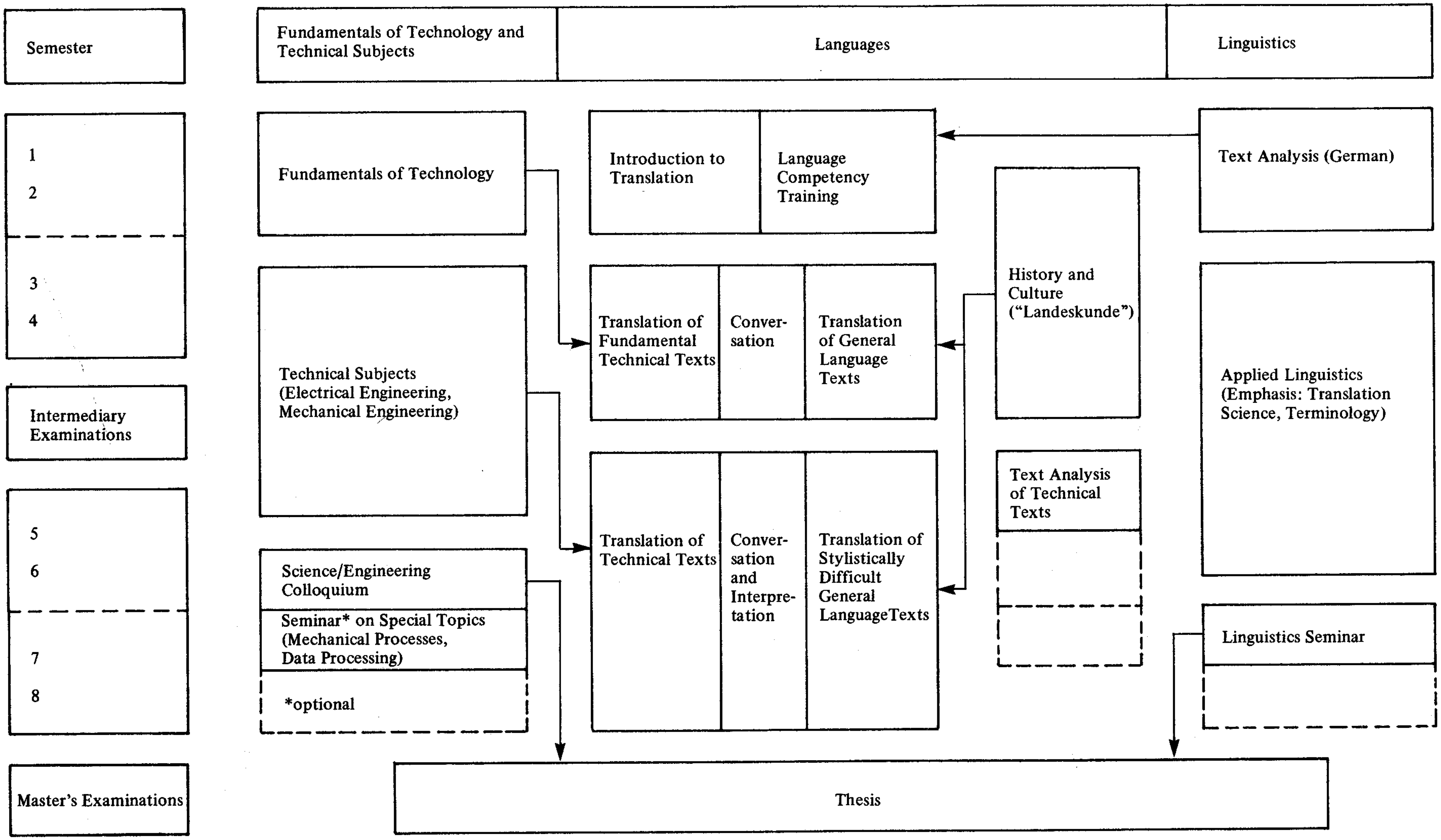
Experiences with «Fundamentals of Technology» and Conclusions Relating to the Overall Aims of «Mathematics for All»

Because our lessons on technology and mathematics were closely related, our major aims had to be these: team-work, co-operation, and the introduction of alternative forms of instruction.

The customary courses in university mathematics were either not suitable or not available, so I had to construct a new mathematics curriculum myself.

Some details about the framework:

- The mathematical qualifications (abilities, knowledge, skills etc.) of our students are extremely varied. Tests which have been carried out show that:
  * knowledge that can be examined is learnt only for examinations (overemphasising the calculation component);
  * the abilities of imagination, estimation, and visualisation are developed to a relatively low level;
  * graphs of functions and related equations of functions are poorly associated with each other;
  * even many students who are very interested in technical subjects lack confidence in their mathematical ability to solve a problem in the technical field.
- It was not appropriate for me to teach pure mathematics, for, as experience shows, linguistically oriented students frequently have a weak mathe-


Semester
Fundamentals of Technology and Technical Subjects
Languages
Linguistics
1
2
3
4
Intermediary Examinations
5
6
7
8
Master's Examinations
Fundamentals of Technology
Introduction to Translation
Language Competency Training
Text Analysis (German)
History and Culture ("Landeskunde")
Technical Subjects (Electrical Engineering, Mechanical Engineering)
Translation of Fundamental Technical Texts
Conver-sation
Translation of General Language Texts
Applied Linguistics (Emphasis: Translation Science, Terminology)
Text Analysis of Technical Texts
Science/Engineering Colloquium
Seminar* on Special Topics (Mechanical Processes, Data Processing)
*optional
Translation of Technical Texts
Conver-sation and Interpre-tation
Translation of Stylistically Difficult General LanguageTexts
Linguistics Seminar
Thesis

matical background, and, correspondingly, have little interest in (purely theoretical) questions in this field. (By the way, the majority of our students are female. This has given rise to very interesting new insights on the topic of women and mathematics, e. g. [10]; our experience up to now has not shown the existence of the attitudes that are usually attributed to women concerning technical things.)

The trend to modern technologies (calculators, computers) appears to be reducing the amount of calculation; therefore it makes no sense to perform calculations without calculators and computers in our curriculum. Furthermore, we cannot exclude the possibility that in the future the relationship between linguistics and computer science as described above may even change attitudes towards mathematics and the learning of mathematics.

There are a lot of important inquiries into the teaching of mathematics and many critical papers have been written which show clearly that the present secondary school mathematics curriculum doesn't achieve either its own goals or those expected by others (e.g. [1, 6, 7]). The remarks made in the section «Selectivity of the School System» by the Organizing Committee of our theme group [5] set us thinking, too.

Our own experiences with the first mathematical courses for technical translators show the same results: the curriculum of traditional school mathematics (that is primarily aimed at formal education and computational skills) doesn't prepare the students for questions requiring comprehension and sense, particularly those involving central mathematical terms which are used in technology as «hand tools.» Students have no idea how to use the disconnected details, the significance of which they do not understand.

A frequent question, Couldn't we do some practical exercises on this subject?» refers clearly to the problem touched on above. Through a «retreat into calculation» perhaps some students believe they come to understand the facts. No wonder this is the case because at school they have been led to make exactly the same assumption. Bauer [2] shows that because only aspects such as «memory,» «cognition,» and occasionally «production,» are tested in school-leaving examinations, these are therefore the very things that are expected in school mathematics.

(By the way, in our course formulae are used occasionally and calculations made, naturally, but only to further consolidate comprehension of formulae and concepts.)

But it would be worse to Renounce mathematics as a substantial part of the core curriculum of general education.»[5] The consequence of this would be that the comprehension of mathematical interrelations would deteriorate further, even more than appears to be the case now.

There is no reason to deny the significance of mathematics for all in the future [6]. On the contrary, it is a very important objective of mathematical education in the future to raise the level of attainment to a higher degree.

How could this be done?

In a recent paper, R. Fischer [4] has argued that «one of the functions of mathematics education within the official educational system should be to contribute to a liberation from mathematics.» This means that mathematics has become independent in view of its richness and wealth of material, which he calls its «second nature.» Men are running the risk that mathematics will control them, and it will be necessary for them to take steps to avoid this possible state of affairs. «Mathematics education can fulfil this new function mainly by emphasizing questions of sense in the classroom and, thus, questions of the relation between men and knowledge.»

I feel this is a very important aspect, as it improves the mathematics curriculum for the teaching of all students. With regard to the suggestions mentioned above and the fact that the content of the curriculum has to be structured in a new way, I see as particularly important the role of fundamental ideas.

Fundamental Ideas

A possible answer to some of our main problems in this theme group «mathematics for all» has to be to point out «fundamental ideas. « The problems of general mathematical education stem from courses which are too full and which contain mathematical topics which are too much atomized. Interpretations of the concept of «fundamental ideas» are varied. My colleagues and I have looked into the role of fundamental ideas in recent years [9].

What is my understanding of fundamental ideas?

I am going to offer you the following approach: to look at fundamental ideas from different points of view, to clarify why and in which way ideas are fundamental. For the purpose of explanation and demonstration, the following complex of questions will be used:

- Is the establishment of fundamental ideas (including the mathematical topics) related to their position within the mathematical context? (e.g., relation to theory, consideration of relevant logical structure and the topic-specific systematical hierarchy, taking into account the training of mathematicians at a university level, «recruitment problems» of up-and-coming university teachers.)
- How applicable, or even better, how usable are the fundamental ideas and what is their significance in practice? (e. g. relation to the real-life situation, to experience, to the-description of the environment, to the demands of school and work.)
- Which interrelations exist between fundamental ideas and educational objectives such as mathematical modelling, argumentation, acquisition of the ability to use heuristic methods?

In the light of these questions, we divided the concepts of fundamental ideas into the following aspects:

* central ideas («guidelines») embodied in mathematical terms and theorems which have importance within the implied framework of a given theory by being the common basis of numerous postulates based on that theory

or through which a hierarchical development can be achieved. These central ideas relate primarily to the theoretical nature of mathematics. They have only a little significance within our teaching of fundamentals of technology. (Central ideas in the sense of didactics are elementary forms of components in mathematical theories.)

* major mathematizing models. These are mathematical ideas (concepts, theorems, methods, etc.) which are useful for explaining important facts of real life or which are suitable to serne as the terminological framework for the mathematical approach to a multiplicity of situations outside mathematics.

* field-specific strategies are central strategies for problem solving, especially for establishing proofs, finding relationships and concept formation in specific fields of mathematics. These strategies can be characterized as being suitable for a variety of different problems in a field.

Examples for major mathematizing models: functions, differential and difference quotients, integrals, differential and difference equations, graphs, Cartesian and polar coordinates, systems of equations and inequalities, vectors, matrices, events, distributions, stochastic variables, chains, boxes, algorithms, . . .

Examples for field -specific strategies: approximation, linearization, analogy between algebra and geometry, geometrization, estimation, special algorithms (e. g. of Gaulb), analogy between plane and spatial facts, transformation, simulation, principle of counting, testing, special theorems (e.g. fundamental theorem of differential and integral calculus), ...

My thesis is that major mathematizing models and field-specific strategies are apt to structure the process of mathematical learning with a lasting effect. They have to involve:

- comprehensive relevance
- sense-creating significance

This is the «higher level» I talked about above before pointing out my views concerning fundamental ideas. R. Fischer [4] suggests that «it is not necessary and furthermore not meaningful for the next generation to learn all those things we have learned.» Indeed, there are a lot of common outcomes which are taught in the present curriculum which are not fun and (I suppose) make no sense any more.

The majority of our future technical translators have not continued taking mathematics in their last secondary years, and there is only a little time to teach some mathematics successfully (within a limit of about 50 hours!). But there are contexts enough in order to teach mathematizing; nearly all facts in our coursework are concerned with fundamentals of technology and technical subjects which include the teaching of mathematizing models.[2] We have seen that, apparently because students were working in a context, they actually became interested in mathematical concepts because they became relevant to their studies.

There are a lot of difficulties when students have to interpret graphs and especially when they have to use the extensive symbolism in mathematics. Interpreting this notation is necessary for grasping the underlying concepts; there is no possibility of avoiding these difficulties because notation occurs in scientific texts. In this respect, the teacher has the task of helping over and over again, and for the very reason that this may take a long time, it is necessary to start at an early age. And, furthermore, it is necessary to demand questions of sense in the mathematical curriculum at all times and in all places.

To teach and to learn the process of concept formation is not easy, but I am convinced that this way is better than the one the present curriculum provides. This is my last message. Perhaps this is a chance to help students lose their widespread fear of mathematics and mathematical education.

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[2] Examples are given in [9].

Part III:
Problems and Developments in Developing Countries

Having a Feel for Calculations*

David W. Carraher, Terezinha N. Carraher, and Analucia D. Schliemann

In studying the arithmetic of Liberian tailors Reed and Lave (1981) proposed that there were two qualitatively different modes of doing arithmetic. The unschooled tailors used a «manipulation of quantities» approach — an oral, context-based way of working with numbers — in contrast to the «manipulation of symbols» approach employed by their schooled counterparts. It is possible that such different modes of doing arithmetic may be found within the same individuals, especially if they use maths in everyday work settings. If so, it could be instructive to look at and compare these modes and to investigate possible relationships between them.

The present report looks at these two ways of understanding and doing maths. We will draw primarily upon a study (Carraher/Carraher/Schliemann, 1982, 1985) which we conducted among young street vendors in northeast Brazil; but we should recognise, if only in passing, the relevance, to our analysis, of cross-cultural studies, particularly those of Gay and Cole (1967), Lave (forthcoming), Lave, Murtaugh, & de La Rocha (1984), Scribner (1984), and Saxe and Posner (1983).

The present study investigated the uses of mathematics by young schooled vendors who use maths in their jobs in the informal sector of the economy (Cavalcanti, 1978.) and who belonged to social classes which characteristically fail in grade school, often for problems in maths. The study proposed to compare and contrast the quality of maths performance among the same children in the market place — the informal setting — and in a formal setting.

In the informal setting, interviewers were customers who made purchases of fruits, vegetables, or popcorn from the vendors. In the course of the transaction they posed questions about real or possible purchases, such as «How much would 6 oranges cost (at 15 cruzeiros each)?» or «How much change will I receive if I pay for the oranges with a 200 cruzeiro bill? « .

The vendors often worked out their calculations spontaneously in an outloud fashion, as in the case below:

Customer: «How much is one coconut?»

Vendor (12 years old,3rd grade): «35.»

Customer: «I'd like 10. How much is that?»

Vendor: «Three will be 105, with 3 more, that will be 210 ... I need 4 more ... that is ... 315 ... I think it is 350.»

* Support for the present research was received from the Conselho Nacional de Desenvolvimento Cientifico e Tecnologico, Brasilia.

In cases where the reasoning was not clear, minimal questioning by the customer was sufficient for the vendor to describe his steps.

In the above case, the question posed by the interviewer may be formally represented as 35 x 10. The child's elaborate procedure consisted in the use of repeated chunked additions for multiplying. The response of the child could be formally represented in the following manner: (3 x 35) + (3 x 35) + (3 x 35) + 35 = 350, where the «chunking» is reflected in the parentheses. Notice also that the vendor must keep track of successive subtotals («I need four more») so he knows when to stop.

We gave five vendors (mean age 11.2 years) who had diverse levels of schooling (from 1 to 8 years) a total of 63 items in the market place. *They answered correctly, without using paper and pencil, in 98.2% of the cases.*

Similar or formally identical problems were devised for testing later in the child's home under conditions which were «formal» in the sense that testing was done with the experimenter and child seated together at a table, paper and pencil before them, engaged in a school-like task. In this situation, word problems which involved calculating with money and computation exercises (with no reference to real objects or money) were given. *The success rates were 73.4% for the word problems and 36.8% for the computation exercises.* A Friedman 2-way ANOVA on ranks showed the performance of the vendors to be significantly different according to condition (p = 0.039).

No less dramatic than the quantitative differences were the qualitative differences in performance depending upon the condition. The following protocol is that of a 12-year-old vendor who, in the market place shortly before, had correctly figured out how much 4 coconuts cost at 35 cruzeiros each.

Interviewer (in home situation): How much is 35 times 4?

Child writes:	*Child* says:
2	«Four times five is 20;carry the
35	two (which is written above the
x4	three). Two plus three is
200	five . . . times four is 20».

(What happened is that the child added the two onto the three before multiplying, rather than after.)

It is instructive to look at other contrasts of this sort. M, aged 11 years, responded correctly and without any appreciable pause when asked in the market place what 6 kilos of watermelon would cost (at 50 cruzeiros per kilo).

Customer: «Let me see. How did you do that so fast?»

Child: «Counting one by one. Two kilos, one hundred. Two hundred. Three hundred».

On the formal test, the child's procedure is different:

Interviewer [reading test item aloud]: «A fisherman caught 50 fish. The second one caught 6 times the amount of fish the first fisherman had caught. How many fish did the lucky fisherman catch?»

Child [writes down 50 x 6, with 360 as the result. Then answers]: «Thirty-six». [The examiner repeats the problem and the child does the computation again, this time recording 860 as the result. His oral response is 86.]

Examiner: «How did you calculate that?»

Child: «I did it like this. Six times six is thirty-six. Then I put it there».

Examiner: « where did you put it?» [Child had not written down the number to be carried.]

Child [pointing to the digit 5 in 50]: «That makes 86». [Apparently adding 3 and 5 and placing this sum in the result.]

Examiner [checking to see that the child had not forgotten the original numbers]: How many fish did the first fisherman catch?»

Child: «Fifty».

Another child, in the market place, is asked to give change for a 500 cruzeiro bill on a 40 cruzeiro purchase. Before reaching for the customer's change he subtracts by adding on: «Eighty, ninety, one hundred. Four hundred and twenty.» In the formal test he must solve the problem «420 plus 80». He misaligns the 8 under the 4 and adds, getting 130 as the answer. Though the reasoning was not made explicit, it appears that the child added the 8 to the 2, failed to lower the zero, but carried the 1, then added the 8 again, this time to the 4 carrying the 1. The child is once again given the problem and proceeds mentally, getting the correct answer.

In sum, then, we are faced with two basic facts: (1) the performance of the vendors in the market place was substantially superior to their performance on problems in the formal setting; (2) the procedures were qualitatively different. Procedures in the formal setting tended to involve written, right-to-left computation. Procedures in the market place were oral and used techniques which did not emerge in the formal setting, such as chunked additions for multiplication problems and subtraction by adding on.

Many issues were raised by these findings, questions such as

- Are the differences in performance a matter of the concreteness or abstractness involved?
- Is the important dimension the coral vs. written» continuum?
- How much can the results be explained on the basis of poor teaching?
- How does school maths relate to this other knowledge of computation displayed by the children?

Some of these issues are addressed by subsequent research which is being reported in this Congress (Carraher, 1984). Here we would like to address, in particular, the issue of concreteness vs. abstractness.

It should be noted that there is nothing inherent to fruits and vegetables which should make the calculations easier. That is to say, there is nothing particularly mathematical about produce. An inspection of the protocols does not give much support to the idea that the vendors had memorised the prices: their pacing and subtotals demonstrates that they work out the problems as they go along. And it should be recalled that they did the problems in their head, without the benefit of pencil and paper for recording intermediary steps.

Perhaps it is not so much a question of relative ease of the market place problems as the relative difficulty of the school problems. But one might ask: Why should arithmetic be particularly difficult (except for the «weak», «dull», or «deprived» child)?

An historical consideration of multiplication shows that what schools teach today as Arithmetic is, in fact, one set of concepts and procedures among several alternatives. In modern Western societies, for example, we learn to multiply by «column multiplication». Unknown to most users of the system, column multiplication is really only one procedure of several which have been invented throughout history. In ancient Egypt, multiplication was performed by a «halving and doubling» procedure (see Diagram 1). During much of the Middle Ages in Europe counters and counting boards were used for multiplying, as well as other computations (Damerow/Lefevre,1981). Roman numerals were used for recording answers but not for actually computing. Even after the introduction of Indian or Arabic numerals and new methods for computing, reckoning boards and counters continued to be used for centuries and there was resistance among many Europeans to learning to work out numerical problems on paper. But even paper and pencil methods varied considerably. In Venice hundreds of years ago complicated lattices were used for multiplying (see Diagram 2).

If we try to understand these systems today, we find them, at least at first, awkward and strange. We begin to understand what it means to say that numeric systems involve arbitrary conventions for the manipulation of symbols. When one uses a computational procedure not fully understood one is likely to make errors through being out of touch with what is going on. Even mathematics educators may have an appreciation for this lack of touch when they try to take square roots or to multiply determinants. For most people, such procedures are not clear, and rote memory must be relied upon. It should be recognised, however, that some school procedures do begin to make sense and one may begin to develop an understanding for what were formerly strange conventions. How does this happen? How is school mathematics related to the informal types of maths described in the present report? What leads to their integration? This remains an important theoretical as well as a practical issue.

The present analysis strongly suggests that the errors which the street vendors make when using school taught procedures do not reflect a lack of understanding of addition, subtraction, and multiplication but rather a difficulty with the system of symbol manipulation conventionally adopted in our societies for solving arithmetic problems. Borrowing, for example, is a typical stumbling block for maths as presently taught in schools. It should be recognised that it is possible to subtract without borrowing, and the vendors do subtract in this way, using regroupings to decompose the problem into one with intermediate steps. There appears to be a gulf between the rich intuitive understanding which these vendors display and the understanding which educators, with good reason, would like to impart or develop. While one could argue that the youngsters are out of touch with the formal systems of notation and numerical operations, it could be argued that the educational system is out of touch with its clientele. Bridging this gap would require, it appears, a better knowledge on the part of educators of the «spontaneous» procedures and concepts which pupils bring into the classroom, or perhaps, leave at the entranceway.

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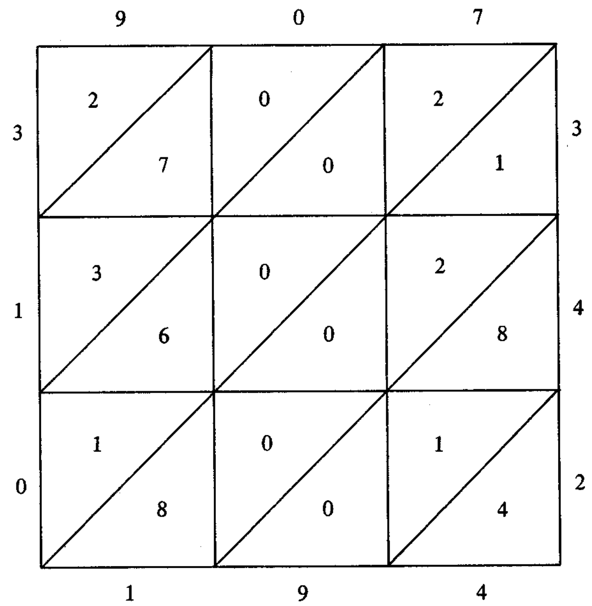
Diagrarn I: The «Halving and Doubling» Method of Multiplying

Example :	13 x 17 ->	17
	6 34	
	3 68 ->	+ 68
	1 136->	+136
	Answer	221

Explanation: The lesser of the multiplicands (13) is successively halved, the result being written in the left-hand column. When there is a remainder (always of one half), it is discarded. In the next column, the multiplicand is successively doubled. Those members of the column which stand opposite odd numbers in the left-hand column are set aside and summed. 17 + 68 + 136 = 221, the correct answer to 13 x 17.

Diagram 2: The Diagonal Lattice Method of Multiplying 907 x 342



Explanation: All the possible multiplications, digit by digit, are made and the results placed in the boxes. Summing proceeds from the bottom right to the bottom left, then upwards to the top left. Each digit of the final answer (310 194) is determined by adding the elements of the corresponding diagonal.

Can Mathematics Teachers Teach Proportions?

Terezinha N. Carraher, David W. Carraher, and
Analucia D. Schliemann

When pupils learn new topics in mathematics in school — say, ratio and proportions — one usually assumes that their problem-solving ability has been expanded. Presumably, they will be able to solve problems which they were previously incapable of solving since they have at their disposal the mathematical knowledge required for proper solution. But how can pupils tell when this (or any) mathematical knowledge is called for? And how do they choose which information to plug into the mathematical routine when there are many (irrelevant) facts available?

If mathematics is to be useful to everyone, mathematics teachers must consider carefully issues related to the transfer of knowledge acquired in the classroom to other problem-solving situations. When pupils learn a general problem-solving procedure in mathematics classes, it is important that teachers concern themselves with ways of turning these procedures into resources that their pupils will, in fact, draw upon when actual problems arise. Following the computational procedures appropriately in the classroom in no way assures that they will be used elsewhere when the lesson is over.

The Rule-of-Three is a simple procedure which mathematics teachers present to pupils as a neat formal way of solving problems involving ratio and proportions. When a is to b as c is to *d,* one can find any unknown from the other three values. The mathematics involved is quite straightforward. However, the simplicity of the mathematics in already-set-up problems may easily mislead one into treating proportions as a topic which can be readily learned by pupils in school. A problem must first be seen as one which calls for a proportionality analysis before the Rule-of-Three is considered a viable approach to its solution. Besides general matters of transfer, cognitive development may be another point to consider; several researchers (Piaget/Inhelder, 1951; Inhelder/Piaget, 1955; Piaget/Grizel/Szeminska/Bang, 1968; Karplus/Peterson, 1970; Aguiar, 1980; Lima, 1982) have shown with different contents that children adopt additive solutions to ratio problems at earlier stages in development and that it is only when the stage of formal operations is reached that proportionality reasoning seems to appear.

In order to better understand how knowledge from mathematics is deployed in other school-related subjects, we looked in this study at how pupils solved three proportionality problems from physics. We also investigated the tendency to use the Rule-of-Three in two conditions: (1) when only the essential information was given; and (2) when relevant information was given along with information irrelevant for solving the problem.

Method

Three problems involving proportions were presented in six different forms each to 720 Brazilian pupils ranging in age from 14 to 20 years and in level of schooling from 6th grade to the last year of high school. This covered a range of six years of schooling, with the lowest level corresponding to the year at which proportions are taught in Brazil. Testing was done collectively in written form and pupils were asked to select one of six alternative answers to each problem and justify their choice. Each pupil received a paper containing one form of each of the three problems; order of problems on the papers followed the Latin Square.

One problem involved judging the height of a building from its shadow, when the height and a shadow of a pole are known. The second involved determining the weight necessary to balance a scale with unequal arm lengths. The third problem type involved the size of shadows as a function of the distance of an object from the light source and the size of the object. Six different forms of presentation of the three problems were designed to check the influence of the following variations upon problem difficulty: (1) verbal versus diagramatic presentation of the problem; (2) specific, numeric versus general, algebraic response form; (3) availability versus non-availability of a formula to compute solution; and (4) applying a computation procedure to given values versus identifying the relevant parameters upon which the procedure should be applied. The first two variations were orthogonal to the three ratio problems; the last two were restricted to one or two specific problems. In all, a total of 18 versions of the problems were used.

Results

Even though the verbal presentation of the problems explicitly mentioned that parameters were directly/inversely proportional — which could have been used by pupils as a cue to the choice of the Rule-of-Three — the form of presentation, verbal versus diagramatic, had no effect upon the difficulty of the items. Further, computing with direct proportions was consistently easier than computing with inverse proportions; success rates varied between 30% and 46% for the direct proportions problems and were around 12% for inverse proportions problems. Success also varied according to an interaction between response form - numeric versus algebraic and problem type — direct versus inverse proportions. Computing a response was easier than indicating a formula when direct proportions were involved while the reverse was true for inverse proportions. The relative ease of inverse proportionality problems seemed to result, however, more from a response bias in the algebraic items than from a better understanding of the problem.

Providing students with a formula for the solution rendered solution significantly more likely but percentages of correct responses still remained under 70.

Identifying relevant parameters proved much more difficult than computing a response for the same problem when the necessary information had already been isolated. Pupils were often unable to indicate the relevant parameters. This type of error cannot usually be observed in mathematics lessons.

The justifications provided by a sample of 220 pupils for their answers (660 in total) were analyzed in order to identify the distinct problem-solving routines used. The overwhelming majority of responses was in fact not justified: pupils either provided a linguistic account of their computations (such as «I multiplied and then divided»), or claimed not to know enough about the content of the problem (such as «I have not yet studied this in physics»), or presented rather vague justifications (e.g., «I followed the logic of the problem»). When an explanation was clearly given, the Rule-of-Three was observed with greater frequency (which varied between 18.6% and 20.6% across the three problems) than any other specific explanation. It was usually associated with successful solution when direct proportions were involved while the reverse was true with respect to inverse proportions.

Several pupils (percentages varied between 7.1 and 32.2 across problems) used a functions approach to the problems (e.g., «If the shadow of the pole is 3 times its height, then the shadow of the building is 3 times its height, and the building is one third the shadow»). This approach often resulted in error because pupils used additive comparisons between the measures (e. g., «The shadow is 6 m longer than the pole, thus the building must be 6 m less than its shadow»). This type of error can be related to trends observed by Piaget and Inhelder (1951) and several others in cognitive development. Some pupils observed still other relations between the measures (e.g., «The shadow is the square of the size of the pole. For the same reason, the building is the square root of the size of its shadow»).
A scalar approach (e. g., «The shadow of the pole is one fourth the shadow of the building. That means that the building is 4 times the height of the pole») to the solution was much less common (percentages varied between 3.9 and 7.1 across problems) than either a functions or a Rule-of-Three approach; it tended to yield correct responses with direct proportions and wrong ones with inverse proportions. In summary, what both the functions and the scalar approaches seem to reflect, in general, is an attempt on the pupils' part to relate one set of two numbers in some way and then transfer this relationship to the second set without much analysis of what type of relationship may in fact hold.

Conclusions

Four main conclusions will be stressed here. First, teachers seem to be somewhat successful in teaching pupils how to use formulas to solve problems and significantly less successful in teaching them how to use the Rule-of-Three. The very low success rates in problems with inverse proportions uncover the pupils' difficulties with this algorithm. Second, analyzing problems and rendering them amenable to solution by using the Rule-of-Three is even more difficult for students. This difficulty appears if pupils must simply point out which information is crucial and also if they must indicate an algebraic formula for solving the problem. This result underlines the issues related to the type of knowledge acquired by pupils in mathematics classes and those related to transfer of training. Third, pupils cannot be said to have truly learned proportions if their competence is restricted to their performance in mathematics lessons. Hart (1981), working with the daily life problem of decreasing quantities in a recipe, found even less indication of transfer than that which was observed in our study. Finally, it is necessary to turn back to the theme of this group's work: If mathematics is to be useful to everyone, issues related to the transfer of knowledge from the classroom to other problem-solving situations must receive a much more systematic treatment both by researchers and teachers.

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Mathematics Among Carpentry Apprentices: Implications for School Teaching

Analucia D. Schliemann

One possible source of children's difficulties when dealing with problem solving at school may lie in the discontinuity between formal school methods and the natural strategies they develop in their daily activities (see Carraher/Carraher/Schliemann, 1985). Alternative proposals to minimize this gap require a deeper analysis of how problem-solving skills relate to specific experiences and how arithmetic training contributes to an improvement in problem-solving ability in and out of school. Scribner (in press) has shown that, compared with students, dairy employees show more variability and more effort-saving strategies when solving problems related to their job activities. Lave (in preparation, see Reed/Lave, 1979), working with Liberian tailors, found that problem-solving procedures are closely related to practical and school experiences: tailors who learn arithmetic in the shop understand the general principles of problem solving but have difficulties with large numbers- those who have been to school can easily deal with large numbers by means of school-taught algorithms but more often make absurd errors that are overlooked. These data provide evidence for the context-specific approach (see The Laboratory of Comparative Human Cognition, in press) and suggest that cognitive skills are closely linked to specific experiences and practice. However, concerning problem-solving abilities, clearer data, such as those gathered by Scribner and Cole (1981) on literacy, are still required. In the present study, carried out in Recife, Brazil, data on problem solving among a group of professional carpenters and a group of carpentry apprentices, with different educational backgrounds, are analysed.

The group of professional carpenters was made up of 12 adults who had had from none to five years of formal schooling. They learned their profession while working as assistants to the owner of the shop, in most cases their own fathers. Their verbal reports suggest that this process of instruction closely followed the pattern described by Greenfield and Lave (1982) for informal education. Naturalistic observation of the daily work of these professionals revealed that arithmetical problem solving often occurs when a customer brings to the carpenter a drawing or a photo of a piece of furniture to be made. The carpenter has then to calculate how much wood he needs to buy and how much he will charge for the finished product. He buys wood from large shops already cut into standard pieces from which parts are to be cut.

The group of carpentry apprentices was composed of 18 adolescents from poor backgrounds, aged 13 to 18 years, who attended a three-year course of instruction in carpentry. All of them were also attending the formal school system and had at least four years of school instruction in mathematics. Naturalistic observation of the activities in the carpentry school revealed that: (a) carpentry apprentices start their practical training by performing simple tasks such as cleaning and polishing; (b) teaching is mostly done by demonstration with few verbal explanations that could help to improve performance in more difficult tasks; (c) after one year of the course apprentices begin to build pieces of furniture; (d) instructions for building each piece are accompanied by a drawing or a three-dimensional model of the piece and by a list of all the parts needed, each one specified in terms of length, width and thickness; (e) wood is available in blocks, from which each part is cut with the aid of powered tools; (f) only at the end of the three-year course are apprentices trained how to make up a list of the parts required for building a particular piece of furniture; (g) parallel to practical training, formal classes on language, arithmetic, geometry and drawing are regularly offered with great emphasis laid from the outset on measurement and how to calculate area and volume.

In this study, in order to analyse how the two groups differ in the way they deal with a problem related to their daily work, each of the carpenters and apprentices was asked to find out how much wood he would need to buy if he were to build five beds like the one shown in a drawing (see Figure 1). They were told that they could use paper and pencil, if they so wished. While they were trying to solve the problem, the examiner talked to them and discussed details of the drawing as well as the steps they followed in order to find a solution. The sessions were tape-recorded and were run in the shops during working hours or, for the apprentices, in a school classroom. An observer took notes, which were used in the analysis, together with tape transcripts and written material produced by the subjects.

Results

Results were analysed in terms of arithmetical operations performed, strategies used to perform operations, dimensions taken into account, and final result.

Tables 1 to 4 show the answers of the first-year apprentices compared to those in their second and third year, and to those of the professional carpenters, in each of the above-mentioned items. Only one apprentice did not attempt to perform the task. Two professional carpenters who had never been to school gave a final answer without explaining how it was obtained. These three cases were not included in the analysis that follows.

As shown in Table 1, more than half of the first-year apprentices preferred to use addition even when multiplication could have been applied as a short cut. Among second- and third-year apprentices, multiplication was used by 70% of the subjects and, among professionals by 90%. The correlation between the level of mastery of carpentry (considering that professionals are at the highest level) and the use of multiplication as opposed to addition, although not very high (Kendall's $p = 0.37$), was very significant ($z = 2.70$, $p = 0.0028$).

The strategies to solve addition and subtraction operations were classified into three categories: (a) mental computation, when the answer was immediately given without the use of paper and pencil; (b) school algorithms, when paper and pencil were used and the answer was found by working initially with units, then with tens, followed by hundreds, and so on, with carrying from one column to the next, whenever needed; (c) mixed strategy, when mental computation was used for the simpler operations and school algorithms for the harder ones. Table 2 shows that mental computation occurred more often among professionals and that school algorithms were preferred by apprentices: 9 out of 10 professionals used head computing in isolation or combined with school algorithms, while only 4 out of 17 apprentices did the same. The Fisher Exact Probability Test shows that such distributions differ significantly ($p = 0.005$).

Table 3 shows that more than half of the first-year apprentices considered only the length of the parts in their attempts to solve the problem. Second- and third-year apprentices considered both length and width or, in most cases, length, width, and thickness. Professional carpenters always worked with the three dimensions. The correlation between the number of dimensions considered and the level of mastery of carpentry was very significant (Kendall's $t = 0.59, Z = 4.13$, $p < 0.0001$).

The final answers given to the question «How much wood do you need to buy if you have to build five beds like the one in the drawing?», were classified into four categories. In the first kind of answer, where 9 apprentices were classified (see Table 4), the dimensions considered were all added up and a final result was inadequately given as the number of meters, or square meters, or cubic meters necessary to build the beds. A second sort of answer, given by 6 apprentices, consisted of a specification of the length, the width and, in some cases, the thickness of a huge block of wood. The length of such a block was obtained by adding up the length of each part of the bed, the width by adding up the width of each part, and the thickness by adding up the thicknesses. The third category of answers consisted of a list of all the parts with a specification of how many of each was needed to build one or five beds. Only 2 second-year apprentices gave such an answer. Finally, the fourth kind of answer, where 8 professionals were classified, consisted of the compilation of two lists, the first specifying the parts as seen in the third category, and the second listing the standard parts usually found in the market from which the parts could be cut. Two professionals, not included in Table 4, after computing the size of certain pieces gave a final answer in terms of how much money the five beds would cost. Correlation between the degree of mastery of carpentry and the kind of answer, considering that the fourth category was the best of all, was very high (Kendall's $t = 0.85$) and significant ($z = 5.95$, $p < 0.0001$)

An analysis of the relationship between the answers given by professional carpenters and the number of years they had been to school did not reveal any clear trend, the only noteworthy feature being that the two subjects who declined to explain how they arrived at a final answer were illiterate.

Discussion

The results obtained in this study suggest first of all that, when faced with a problem-solving task, individuals try to find an answer that is closely related to their daily experience: while professional carpenters seek a list of standard pieces to buy, apprentices try to find the measures of a block of wood from which parts could be cut. What is most striking in these attempts is the suitability of professional carpenters' strategies to find a solution when compared with the unsuitability of the apprentices' approach. Although the apprentices had had formal teaching on how to calculate volume, their attempts were unsuccessful and the results obtained were absurd. However, they did not seem to perceive the absurdity. It seems that the task was approached by the apprentices as a school assignment and they did not try to judge the suitability of the answers. For the professionals it was taken as a practical assignment and the solution sought was a feasible one. That difference between a school approach and a practical approach, as noted by Lave (personal communication), seems to change the nature of the problem.

Computing strategies, although different, were equally effective in both groups, for hardly any mistakes were made. This is an unexpected result bearing in mind that formal school attendance was very different between the two groups and among the individuals in the professional group.

Of special importance for education is the fact that, despite receiving special teaching on how to calculate area and volume, and how to solve formal problems involving these, apprentices were not able to use this formal knowledge to solve a practical problem. This fact is even more striking if we consider that the elements of the problem were part of their daily experience. It seems then that problem solving at school has to be taught differently if it is to have any use out of school. One possible suggestion arising from the data presented here is to provide, in addition to formal teaching, opportunities for problem solving in practical contexts. This may improve comprehension and lead to the discovery of new and more economical strategies and solutions.

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Table 1: Number of Subjects in Each Sub-group According to Operations Used While Trying to Solve the Problem

Sub-groups	Addition	Addition and Multi-plication	Total
1 st-year Apprentices	4	3	7
2nd- and 3rd-year Apprentices	3	7	10
Professional Carpenters	1	9	10

Table 2: Number of Subjects in Each Sub-group According to Strategies Used to Solve Arithmetical Operations

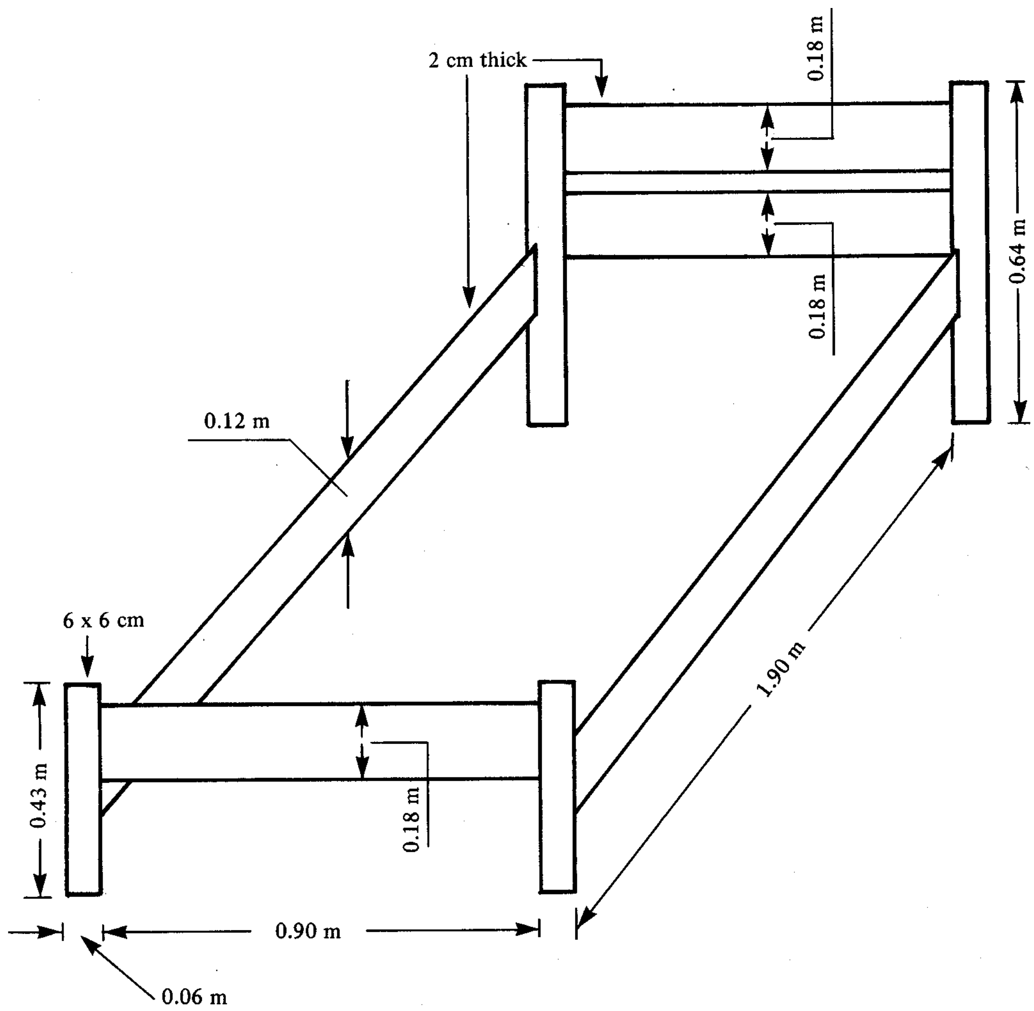
Sub-groups	Mental	Mixed Computation	School Strategy	Total Algorithms
Ist-year Apprentices	0	1	6	7
2nd - and 3rd - year Apprentices	0	3	7	10
Professional Carpenters	0	0	10	10

Table 3: Number of Subjects in Each Sub -group According to Dimensions Considered When Trying to Solve the Problem

Sub-groups	Length	Length and Width	Length, Width and Thickness	Total
Ist-year Apprentices	4	1	2	7
2nd- and 3rd-year Apprentices	0	4	6	10
Professional Carpenters	0	0	10	10

Table 4: Number of Subjects in Each Sub -group According to Kind of Final Answer Given

Sub-group	Addition of all dimensions considered	Block from adding up each dimension	List of parts	List of standard parts
Ist-year Apprentices	6	1	0	0
2nd- and 3rd-year Apprentices	3	5	2	0
Professional Carpenters	0	0	0	8


2 cm thick
0.18 m
0.18 m
0.64 m
0.12 m
6 x 6 cm
0.43 m
0.18 m
1.90 m
0.90 m
0.06 m

The Relevance of Primary School Mathematics in Tribal Aboriginal Communities*

Pam Harris

Introduction

Up until fairly recently (i. e. within the last ten years) it has usually been the policy of Departments of Education to expect all schools to use the same mathematics syllabus in basically the same way, irrespective of the cultural, linguistic, and mathematical background of the pupils. In Aboriginal schools in the Northern Territory this led eventually to a fairly strong backlash from teachers who claimed that wit is not relevant to our situation», the Bite referring to either mathematics in general, or to the particular syllabus they were required to follow, and specific requirements of that syllabus.

Relevance of curriculum content has long been an issue in Aboriginal schools and is bound to remain so for as long as Aboriginal people lack control over the education their children receive, and non-Aboriginal teachers come from outside to teach in a situation they do not understand. Some education administrators are inclined to dismiss the issue, saying that teachers are using irrelevance as an excuse for their own laziness and incompetence, and that good teaching of the set syllabus without adjustments is all that is needed for Aboriginal pupils to succeed in maths.

Whilst it is no doubt true that some teachers may too easily give up on teaching maths, rationalising that it is «not relevant» to their Aboriginal pupils so they Won't do its, the question cannot be so easily dismissed. Many experienced and hard-working teachers have questioned the relevance of the mathematics curriculum in Aboriginal communities in such comments as «You do your best, but you wonder what it's all for», «It's not worthwhile to teach them what they're not going to use», and «What's the use of teaching topics that are not needed?».

The question of whether the same syllabus and set of aims for teaching primary mathematics is equally suitable for both Aboriginal and non-Aboriginal schools—and whether it would even be practicable to have different guidelines—is one which is decided separately in each state department, and it is not proposed to discuss it here. In this paper I will look at the broader question of whether primary mathematics is relevant for tribal Aboriginal children living in tradition-oriented communities.

First I will outline some of the factors which discourage people who are teaching maths in remote Aboriginal communities and which often lead to the protests that it is not relevant. Then I will consider factors that make mathematics a particularly difficult subject, not just for one group, such as Aborigines, but for many people in any population. That should help to put the problems frequently encountered in Aboriginal schools into perspective. Finally I will give reasons why maths *is* relevant in Aboriginal communities, though the aspects which are most relevant and useful may be different from those which are most obviously relevant in other types of communities.

1. The Feeling that Mathematics is Not Relevant in Aboriginal Communities — How it Arises

The feeling which some teachers have that maths — or most of it — is not relevant in the remote Aboriginal communities where they are teaching seems to come from three main sources:

1. Negative expectations passed on by other people.
2. The teacher's own observations of lack of reinforcement of maths in the pupils' home life.
3. The cultural and linguistic bias of teaching materials.
4. Discouragement because of difficulties teaching maths and the pupils' generally low level of achievement.

If these influences on the classroom teacher's attitude can be appreciated by mathematics advisers and those controlling curriculum decisions, and if the teachers themselves can understand some of the forces at work, this should help to put the question of relevance into perspective and enable freer communication between all levels of those concerned with primary maths education in Aboriginal schools.

1.1 Negative expectations of teachers

It has been reported previously (P. Harris, 1980, p. 19) how teachers often receive negative attitudes from other people to the extent that they go to an Aboriginal community expecting that their pupils will not be able to do mathematics. A stereotype very common in the wider Australian community is that «Aborigines can only count one, two, three, many» followed by the conclusion, self-evident to the speaker, that therefore they cannot do maths. This stereotype is based on a mixture of fact, ignorance, and over -generalisation.

The fact is that most Aboriginal languages do have very few words for cardinal number. It is common to have separate words only for one and two, and perhaps three, and then words that refer to a few and many, or, as it is often colloquially said «little mob» and «big mob». However, there is ignorance about what this really means for Aboriginal children learning to count (which they most often

* This paper is part of a larger publication entitled Teaching Mathematics in Tribal Aboriginal Schools, which is one of four publications in the Mathematics in Aboriginal Schools Project series. The Mathematics in Aboriginal Schools Project was a national research project jointly funded by the Curriculum Development Centre in Canberra and the Northern Territory Department of Education during 1980-81.

do in English), and it is a gross over-generalisation to assume that some lack of counting vocabulary in their own language could be taken as evidence of a general lack of ability to cope with all areas of mathematics. (The lack of number words in Aboriginal languages has been briefly discussed in P. Harris, 1980, p. 13.)

Contrary to the popular myths, one prominent linguist who is familiar with Australian languages has argued that the gap in number vocabulary does not indicate that counting itself is lacking in the culture. He suggests that it is there «in the sense that the principle of addition which underlies the activity of exact enumeration is everywhere present» (Hale, n. d.).

Although new teachers going to Aboriginal schools may not consciously espouse the common myths and stereotypes, they are often confronted with them and, without any information to the contrary, it is not surprising that many of the teachers take up their new appointments with low expectations of what their pupils might achieve in mathematics, and a feeling that there is probably not a great deal that they personally can do to change the situation.

1.2 Lack of reinforcement of maths in the pupils' home life

On arriving in an Aboriginal community, the teacher is often greatly impressed by the difference in lifestyle and living conditions — differences which they see have implications for their teaching of mathematics in that place.

Living conditions vary of course, and there are some communities where the pupils are living in conditions not too different from the teachers', but in many of the more distant tradition-oriented communities with which the Mathematics in Aboriginal Schools Project has been particularly concerned, the teacher will find that their pupils live in makeshift humpies, beach camps, one-roomed tin huts, or maybe in nothing at all, just sleeping behind a windbreak. This reality comes as a shock to some.

As the teacher begins to see the many aspects of mathematics which do not appear to be reinforced in the pupils' home life—aspects which are constantly reinforced in the home lives of most Anglo-Australian children—the word «irrelevant» soon comes to mind. It does not seem relevant, for example, to teach young children to tell the time on a clock when the teacher knows that very few, if any, of the pupils have a clock in their home, their parents rarely mention clock time, and, in fact, about the only time they see a clock or are expected to use one is in the classroom. Telling the time seems to be a school-based activity with neither reinforcement nor usefulness in the child's home life,** in contrast to the situation of most children brought up in the Western-European tradition where the child has usually been surrounded by clocks and had its daily activities regulated by the clock almost since it was born, and where the parents often consciously encourage the child or infants of lower primary age to learn to tell the time, and assist it in its efforts, thus reinforcing what the teacher does at school.

Many more examples could be given of areas of mathematics where the non-Aboriginal teacher is often frustrated in attempts to teach skills and knowledge in contexts that will be meaningful to the child and will be used and reinforced outside of the classroom. Fractions, for example, are difficult to teach, and appear to be rarely used, even in employment; and how meaningful and motivating is it to teach measurement of mass (weighing) and measurement of capacity through cooking activities using written recipes, when the child never sees a recipe being followed or a standard measuring instrument being used at home? How does the teacher teach division in a meaningful context when the pupils and their families customarily divide and share in unequal portions according to kinship obligations? These and many more such questions daily confront the teacher and require decisions which the newcomer may not feel qualified to make, not yet having had a chance to think through his or her own philosophy of education in a bicultural situation.

Apart from the lack of home reinforcement during the child's schooling, there is often also an absence of the motivation which some people have to learn mathematics because of its uses in employment. In many communities, both teachers and pupils are well aware that school leavers have little chance of finding a good job — or any kind of job at all. For example, the magazine of one large Aboriginal community, in reporting that a certain young man had started work with the Housing Association, noted that this was his first job since leaving school five years before, and that out of all the young men who had left school over the past six years, only six were employed at the moment (Junga Yimi 2 :3).

1.3 Cultural and linguistic bias of teaching materials

Some teachers, sensitive to the different lifestyle, interest, and aspirations of their Aboriginal pupils, also consider many of the commercially available teaching aids unsuitable. The pictures and examples given in work-books often seem to portray very little that is a familiar part of the Aboriginal child's daily life. And of the hundreds of rhymes and songs available to introduce number and counting to preschoolers and infants, the majority seem to talk about subjects that are rather meaningless to Aboriginal children. While it is possible for individual teachers to overcome this problem to some extent by making their own worksheets, using materials in the environment, and adapting the wording of the number rhymes, the fact that these adaptations are necessary can be a nagging reminder that the maths materials were not intended for Aboriginal pupils.

** Notice that these examples are of things that *seem* irrelevant to many teachers coming into Aboriginal communities — the validity of this conclusion will be discussed later. To follow up the question of teaching clock time in Aboriginal schools, the reader is referred to a publication in the Mathematics in Aboriginal Schools Project series entitled *Teaching about Time in Tribal Aboriginal Communities,* Pam Harris, 1984 published by the Northern Territory Department of Education, Darwin.

The fact that all materials are presented only in the English language seems to be another indication that mathematics is strictly «whitefella» business, not a part of Aboriginal culture or current lifestyle.

1.4 Beaching difficulties and low level of pupil performance
Whether the teacher comes with low or high expectations of their pupils, they are often discouraged by the apparent low level of performance of many of their pupils in mathematics, and the reports they have heard seem to be confirmed. The pupils often do not seem to understand or remember certain things no matter how many times they are taught or how clear the explanation seems to be.

Some teachers who have previously taught English-speaking children may find that the methods that worked before do not seem to work with their Aboriginal pupils, and so may tend to think that the problem lies with the pupils, perhaps with some difference in thinking processes which prevents Aborigines from «catching on» to maths. Other teachers may have tried hard to bridge the gap between the home experiences of their Aboriginal pupils and the background experiences that seem to be assumed in the maths syllabus, to «make mathematics more relevant in the home and community» as suggested in the Northern Territory's 1974 Infants Curriculum, but find they are fighting an uphill battle with little support. In such circumstances it is easy to come to a general conclusion, consciously or unconsciously, that the children in that and similar Aboriginal communities «can't do maths» mainly because «it's irrelevant» — the mismatch between the syllabus requirements and the community's requirements seems to be too great.

Before looking at the counter arguments that the study of mathematics *is* relevant in Aboriginal communities, we should first question the extent to which the difficulties in teaching and learning mathematics which are often experienced in Aboriginal schools are actually peculiar to those schools.

2. The Problem in Perspective

2.1 Mathematics is a difficult subject
The fact is that a great many people everywhere, including many living in sophisticated Western societies, find mathematics much more difficult than other subjects, question its relevance for themselves, dislike it, and feel that they «can't do it».

This frequent rejection of mathematics by otherwise well educated people has been pointed out (and accepted) by leading mathematics writers and mathematicians. The first sentence in Skemp's *Psychology of Learning Mathematics* (1971) talks about «Readers for whom mathematics at school was a collection of unintelligible rules (. . .)», and Kline (1962) begins his *Mathematics: A Cultural Approach* with the words «One can wisely doubt whether the study of mathematics is worth-while (...)». And the great French mathematician René Descartes (1596 — 1650) told how, after doing some study in Arithmetic and Geometry, he found the «hows» and «whys» of the subjects not sufficiently clear, and consequently «was not surprised that many people, even of talent and scholarship, should (...) have either given them up as being empty and childish or, taking them to be difficult and intricate, been deterred at the very outset from learning them (. . .)» (from leading quotation in Kline,1953, *Mathematics in Western Culture).*

There are a number of factors which make mathematics more difficult than other subjects for both school children and adults, and these factors apply just as much for Anglo-Australians and other Westerners as they do for Aborigines.

(1) Mathematics is very abstract, much more abstract than any other subject introduced in the primary school.
(2) Mathematics is more sequential than other subjects.
(3) Mathematics learning is more teacher dependent than other subjects — there is not so much that can be «discovered» by the student working alone.
(4) Mathematics is often taught in a dull, uninteresting way without any meaningful context or examples.
(5) In some areas of mathematics, especially number work, it is possible to perform well without the understanding that will enable the learning to be used later; thus problems are often not detected by the teacher.
(6) There is less support for remedial work in mathematics (e.g. compared to the facilities provided for remedial reading).
(7) There are more teachers who lack confidence in their own grasp of the subject and their ability to teach it than there are in the other basic subjects. (For example, an article in the Arithmetic Teacher, May 1981, states that in one teacher training program in Ohio, two-thirds of the students counted over a nine-year period have named mathematics as their least favourite and most feared subject.)

In addition to these, there are two more major factors which affect the teaching and learning of mathematics in traditional Aboriginal communities.

(8) Learning mathematics and adopting a mathematical way of thinking is like learning and adopting a second culture, and,
(9) when this is done in English, then the second culture has to be learned in a second or foreign language.

These last two points need some further explanation.

2.2 Learning Mathematics is like learning another culture
When an immigrant child, whose family speaks for example, only Greek at home, enters an Australian primary school and is required to learn mathematics in English, this is not as difficult a task as when a vernacular-speaking Aboriginal child is required to learn mathematics in English.

The Greek child is already part of a culture which has a rich tradition of mathematics going back for hundreds and thousands of years. It is part of that child's way of life and way of thinking, and the child's task on entering the English-speaking program is merely to transfer what he or she *already knows* from his or her own language into another language which is closely related.

The Aboriginal child's task is different, and much more difficult. He also has a rich cultural heritage, but it does not include much of the Western mathematics which is taught in primary schools. Western mathematics is a new way of thinking, a new way of ordering the world which is in many respects at variance with Aboriginal ways. In a sense, it is another culture. For the Aboriginal student, learning mathematics in English is not a case of transferring ideas from one language to another; old ideas must be reorganised and a whole range of new ideas must be learned and appreciated.

2.3 Aboriginal children often have to learn the «second culture « of mathematics in a second or foreign language

Many Aboriginal children have to learn the second culture in a second or foreign language - English. A N. T. Department of Education linguist, who is herself bilingual in English and French, is of the opinion that number and mathematics are among the most difficult areas of learning in a second language. This applies even to «those of us who have all the conceptual knowledge at our fingertips» (Mary Laughren, pers. comm.). Ways of expressing mathematical ideas such as comparison are very language specific and the differences between languages are great, even between closely related languages such as French and English.

If these language differences and learning difficulties are so significant for highly educated people who have a similar mathematical background and speak a closely related language, how much more significant and potentially constraining must they be for Aboriginal people whose mathematical background is quite different and whose own language is quite unrelated to English or any of the other languages which have contributed to the growth of formal mathematics, such as Greek, Hindu, and Arabic?

Having looked at some of the differences which impress and often discourage those involved in mathematics education in tradition-oriented Aboriginal communities, and tried to put them into clearer perspective, we now turn to look at the positive side — the assertion that mathematics is relevant in Aboriginal communities.

3. The Assertion that Mathematics is Relevant in Aboriginal Schools

3.1 Mathematics is relevant because . . .

Mathematics is relevant and necessary in tradition-oriented Aboriginal communities as it is in other Australian communities, for the following reasons —

3.1.1 Mathematics is needed in everyday life, in employment, and in the conduct of community affairs

People tend to think that the more Aborigines move back to their homelands and assert their right to live in an Aboriginal way, as many are doing these days, the less they will need or want what Western-style education provides, including mathematics. The practical reality is exactly the opposite. In order for an Aboriginal community to exist independently and run its own affairs according to the wishes of its people, there must be at least some in the group who are fluent in English and competent in mathematics and thus able to communicate confidently with government officials and other white Australians in the wider community. The greater the desire for independence, the more urgent is the need for Aboriginal people to acquire for themselves skills in English literacy and Western mathematics.

(By stressing the need for skills in Western mathematics and literacy in English in this context, I am not at all questioning the value of bilingual/bicultural education programs in which the early emphasis is on acquiring literacy in the vernacular and understanding mathematics concepts which are a part of the traditional tribal way of life. These vernacular programs, apart from their other advantages, provide a sound basis for improved performance when the student must later tackle English literacy and primary school mathematics taught through the medium of English as a second language.)

3.1.2 Aboriginal people have requested mathematics

Whenever tribal Aborigines have stated what they want from education, they have always (in my experience) included high on the list of priorities (a) ability to speak, read, and write English, and (b) Knowing numbers The reasons given for wanting to «know numbers» are very practically oriented to managing their own affairs. See, for example, comments recorded by H. H. Penny in the report of his investigation into the training of Pitjantjatjara teachers in South Australia (1976, p. 18).

3.1.3 Mathematics is necessary for secondary and most tertiary education

If Aborigines are to achieve their aims of being teachers, doctors, mechanics, etc. then they must have a good basic understanding of maths, and the option to choose to do it at higher levels beyond primary schooling.

3.1.4 Mathematics is a major clue to understanding the way Anglo-Australians think (in line with their Western-European cultural heritage)

Morris Kline begins his important book *Mathematics in Western Culture* thus:

«(.. .) mathematics has been a major cultural force in Western civilisation. Almost everyone knows that mathematics services the very practical purpose of dictating engineering design (v.) It is (...) less widely known that mathematics has determined the direction and content of much philosophic thought, has destroyed and rebuilt religious doctrines, has supplied substance and economic and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universes (Kline,1953)

To understand the Anglo-Australian culture by which they are surrounded, Aboriginal people need to have some understanding of mathematical thinking.

3.2 Some mathematics topics are more relevant than others

Nevertheless, although mathematics as an area of study is relevant in both Aboriginal and non-Aboriginal societies, it soon becomes evident to the teacher in an outback Aboriginal community that some maths topics (like money, for example) are much *more relevant* than others, some which are accepted without question in other schools appear to have *very little relevance* (e. g. fractions, division), and yet others appear to be *relevant and motivating only if they are introduced at a different stage from that recommended in the syllabus* and with different emphasis (examples are the introduction of standard units of measure and learning how to tell the time on a clock).

The teacher soon sees a need to adjust the syllabus to meet local requirements. This does not imply that the aims for the endpoints to be reached by the end of primary school should be changed, but that in Aboriginal schools these endpoints may be more effectively reached through a primary maths program that has different emphases, different sequencing, and different teaching methods from those recommended for Anglo-Australian children.

3.3 Some mathematics topics are more useful than they at first appear

Adjustment to the syllabus are necessary to meet the needs of differing local conditions, but newcomers especially should be wary of making changes, particularly any which involve not treating a topic because it seems irrelevant. Such decisions need to be made only after careful consideration of the future needs of the child and the relation of that topic to other parts of the syllabus, and are best made in consultation with an adviser, where one is available.

3.3.1 For example, why teach fractions?

One Northern Territory teacher, an experienced and conscientious person, once wrote and asked the mathematics curriculum unit in Darwin to give her some good reasons why she should teach fractions, because she said she could not see the use of them and there seemed to her to be more important things on which to spend one's teaching time.

The questions «Why teach fractions?» and «Should we teach fractions at all?» are often asked in Aboriginal schools, so I will give here some reasons for teaching fractions and these will serve as an example of the various aspects to be considered in regard to topics which at first seem irrelevant.

Aboriginal children, like any others, need an elementary understanding of fractions because:

(1) Fraction terms such as «half» and «quarter» are an integral part of everyday English speech.
(2) Decimal fractions cannot be properly understood if the idea of fractions (equal parts) is not understood.
(3) Fractions are used in employment, for example in the hospital (half dose of medicine for a child), when making out time sheets (time-and-a-half pay for working after hours), and when stock-taking in the store.
(4) Common fractions cannot be entirely replaced by decimals — not all situations involving fractions can be handled in decimal form.

In addition, work on equivalence of fractions and simple addition and subtraction of fractions provides older students with extra practice in the four operations which does not look like the same old «sums» being dished up yet again. That is important for slower students who have not achieved in the four operations but are not motivated by the methods used with younger pupils. One principal in a large Aboriginal secondary boys' school reported that he had found work on fractions very helpful in increasing students' skills in the four operations. This idea is also supported in some teachers' guides, see for example page 152 of *Mathematics - A Way of Thinking* by Robert Baratta-Lorton. Here I have presented linguistic, cultural, practical, mathematical and motivational reasons for retaining some work on fractions. These are just some of the aspects which will have to be considered in every question of adjusting the content and sequencing of the mathematics syllabus to suit a particular situation or group.

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Bicultural Teacher Training in Mathematics Education for Aboriginal Trainees from Traditional Communities*

Kathryn Crawford

Introduction

This paper will discuss the challenges facing an educator in the development of a bicultural, bilingual teacher training programme in mathematics curriculum for Aborigines from traditional communities in Central Australia.

Although many of these challenges stem in particular from the characteristics of the communities involved and their particular culture, there are also many aspects of this educational task that are paralled in any country where efforts are made to cater for indigenous groups of people in an education system that has been derived from Anglo-European cultures.

The course described forms part of the Anangu Teacher Education Programme (ANTEP) an accredited teacher training course intended for traditionally oriented Aboriginal people currently residing in the Anangu communities who wish to take on greater teaching responsibilities in South Australian Anangu schools. The course will be directed from the South Australian C.A.E. but most teaching will be carried out on site by a lecturer residing within the communities. Pukatja (Ernabella) will be the host community for the project.

The programme as a whole represents a significant departure from conventional teacher education in a number of ways. Perhaps the most striking difference between this teacher training course and many others is that from the beginning, development of the curriculum has been a co-operative venture between lecturers and educators on the one hand, and community leaders and prospective students on the other.

The extent of this co-operation is indicated by French-Kennedy's (1984) description of the aims of the curriculum design workshop held in April 1984:

«The general aim (...) was to bring together prospective ANTEP students; interested Anangu; non-Anangu with demonstrated expertise in the area; the relevant ANTEP lecturers in charge and the on-site lecturer for the purpose of considering, in detail, the initial offering of units. « (p. 3)

The first group of students will commence the course in August 1984.

The tone of the early negotiations with the Anangu communities indicated that an interactionist perspective on bicultural education and on mathematics education and curriculum development in particular would be most appropriate. The rationale for the design of the two units Teaching Mathematics I and II that form the mathematical component of the course has evolved from the urgent need to provide experiences that will enable students to negotiate the complex interacting factors from the known in their own culture to a competence in the use of mathematical ideas from Anglo-European cultures. The perceived community needs in Western mathematics were eloquently stated by one member of the community as follows:

«Our children need to know enough maths so they don't get ripped off.»

Initial discussions with community leaders and prospective students suggested the following general aims for the course:

1. Development of student awareness of their cultural expertise in
   a) the Anangu ways of thinking about relationships and patterns to do with the locations, qualities and quantities of objects and people in the environment;
   b) the needs of the Anangu community to
      - affirm Anangu culture and Pitjantjatjara language;
      - develop new strategies and mathematical knowledge to meet the need for dealing with Anglo-Europeans and their culture;
      - explore traditional ways of teaching young children and the modification of these methods as necessary to accommodate new knowledge.
2. Widen student awareness of and ability to apply elementary mathematical knowledge (S.A. Curriculum K—8) to solve community problems.

To enable students to develop a rationale for teaching behaviour and methods that are appropriate to the needs of the children of the community.

Negotiating Meanings Between Two Cultures

Gay and Cole (1967) examine the teaching of mathematics in a cross-cultural situation. They suggest:

«(...) in order to teach mathematics effectively, we must know more about our students. In particular we must know about the indigenous mathematics so that we can build effective bridges to the new mathematics we are trying to introduce.» (p. 1)

The need to build conceptual bridges from the known to the unknown is not of course an educational problem restricted to the context of bicultural education. The mathematics curricula in most primary and secondary schools are notably dissociated from the everyday concerns of the student population. This has been a cause for considerable concern among mathematics educators in an increasingly technological society. In a bicultural context the situation is made more serious by the fact that different cultures emphasise different conceptual schema. Thus, temporal sequences and quantitative measurement are dominant

* This is a revised version of a paper presented at the 1984 Conference of the South Pacific Association of Teacher Education.

themes in industrialised Western cultures but largely irrelevant in traditional Aboriginal cultures. Scientific and technological thought, the appropriate registers of language and mathematics as an abstract discipline have developed over hundreds of years within Anglo-European culture and reflect these dominant themes particularly at the elementary level. Experience suggests that for many Aboriginals from traditional communities the content of the elementary mathematics curriculum is perceived as incomprehensible and often irrelevant.

There has been a great deal of valuable descriptive research by J. Harris (1979), Dasen (1970), Kearins (1976), Rudder (1983) and others about the kinds of classification systems arid the rate and order of development of concepts related to mathematics in differing Aboriginal cultures. The work of Parm Harris[1] provides an account of the effect of these differing cultural perspectives on mathematics learning. Her detailed and historical accounts of Aboriginal attitudes and beliefs about such topics as money, measurement and number provide a valuable resource for teachers. Her work is important in combatting the negative expectations expressed by teachers in such statements as: Aboriginal children do not generalised

The clear accounts of how Aboriginal children *do* generalise and why they find school mathematics difficult are instructive. Her explanations of how these difficulties may be overcome redirect the focus of the problem from the «failings» of Aboriginals and Aboriginal culture to the inappropriateness of many teaching practices for children from traditionally oriented communities.

Most recently published curriculum materials such as those devised by Western (1979), Northern Territory Department of Education (1982) and Guy (1982), intended for use by Aboriginal teacher trainees or as resources for teachers in Aboriginal schools have acknowledged the implications of J. Harris's (1979) statement:

«As the child matures learning to label and order his experiences it is inevitable that his cognitive development will be very strongly influenced by Aboriginal systems of knowledge.» (p. 143)

Thus, these materials take care to acknowledge language difficulties, use materials from a familiar context for illustration and take particular care in topics such as time and measurement to provide experiences to facilitate conceptual development. However, a closer analysis of such materials suggests that the teaching procedures and the content are still culturally biased to the extent that Aboriginal people are likely to have difficulty relating school experiences in mathematics to community needs and problems.

In a community-based teacher training course it seems that it is possible for the first time to develop procedures for negotiating meanings between the two cultures. With this in mind the lecturer's notes at the beginning of the first module in the course state:

«A co-operative exchange of knowledge is particularly important in the tutorial sessions because mathematics by its nature involves the use of higher order cognitive skills and problem solving requires confidence. There is considerable research evidence to suggest that egalitarian relationships foster these skins better than authoritarian directions. It is important that student/teacher interaction and role play used in the sessions provide a suitable model for interaction with children.» (Teaching Mathematics I, Module 1)

The course has been developed based on a model designed to maximise the possibility of interaction between the world view expressed by Anangu culture and that of Anglo-European culture as evidenced in school mathematics. This is achieved by placing an emphasis on the student expertise *and* contribution in providing information about Anangu world views *as a necessary part of the course.* The course is constructed in such a way that students are invited to participate in co-operative decision-making about appropriate methods for negotiating meanings from one culture to another.

Group interaction and co-operation in arriving at solutions to the problems set by tasks in each module are an essential component of course experiences. This is a necessary process if an understood social concensus about the multiple realities, perceived by Anangu and non-Anangu tutorial group members is to be achieved. Such cognitive interaction is the basis for the development of a synthesis of world views and of the clearly understood and generalised universal premises that will form the basis for future group decision-making. The outcomes of such a synthesis are in the changed perspectives that the participants take away with them.

Figure I below illustrates some of the contexts in which this approach will be used.

It is expected that in Teaching Mathematics I the mathematical content will be that of the South Australian Department of Education Syllabus's Early Childhood section (Modules 1 — 10). In the second year of the course the emphasis will shift to the mathematical content of the upper grades of primary school. The community has expressed the desire that teacher trainees should function as a resource of useful mathematical knowledge within the community. To meet this need there will be a mathematics component in the Work Skills Unit that is also included in the ANTEP programme.

[1] Personal correspondence and CDC Mathematics in Aboriginal Schools Project Series (in press).

	Context	Problem-solving Strategies	Language Use	Mathematical Content	Teaching Strategies For generalisation and concept development	 For rote learning of conventions
ANANGU (Students contribution)	Self group survival and cultural activities.	Anangu ways of solving problems related to: classification, space, quantities, measurement.	Pitjantjatjara vocabulary and syntax development to be decided by students.	Increase student awareness of mathematical ideas in Anangu culture. Explore needs for different knowledge and ideas to solve problems arising from wider experience including Anglo-European contact.	Begin with Anangu learning methods and important cultural themes.	Begin with Anangu learning methods.
Group decision-making	Cultural interface.	Used as a basis for discussion of:		Methods for bridging cultural differences.	Compare and contrast with different methods that are important in individualised society.	Compare and contrast with course outcomes.
ANGLO-EUROPEAN (on-site lecturer and resources contribution)	Wider contexts including: Anglo-European contact – trade – employment, etc.	Anglo-European ways of solving problems in a more complex and industrialised culture.	Use of appropriate English vocabulary as necessary and appropriate.	1. Early Childhood curriculum content. Early Primary. 2. Middle grades 5–8 curriculum content. S.A. curriculum.	1. Establish relationships or patterns (use visual input as appropriate). 2. Change contexts. 3. Change quantities (if appropriate). 4. Develop links with other related concepts. 5. Use the new knowledge to solve problems.	1. Use of song, rhythm and movement. 2. Practice. 3. School experience in using skills.

A system of *clustered* modules has been used in the course so that conceptual construction links between related topics are made as explicit as possible.

The diagram below illustrates the first 20 modules of Teaching Mathematics I. See appendix for a more detailed description of the form of a particular module.

Figure 2

Module 1
What is mathematics?
Anangu needs and problems in mathematics

Classification
Modules 2–5

2. Classification and Grouping 1 – Anangu way.
3. Classification and Grouping 2 – according to position.
4. Classification and Grouping 3 – size.
5. Teaching methodologies for classification.

Comparison and Ordering
Modules 6–10

6. Comparison 1 – feeling objects.
7. Comparing people – ordering and posture.
8. Ordering objects and events.
9. Comparing and ordering groups.
10. Developing teaching methods for comparing and ordering.

Early Ideas of Number
Modules 11–15

11. Anangu terms for quantity – uses.
12. 1:1 matching – showing more-same-less.
13. Equivalence – exchange games – trading.
14. Using grouping to describe quantities < 10.
15. Counting to 10. Counting by 10's.

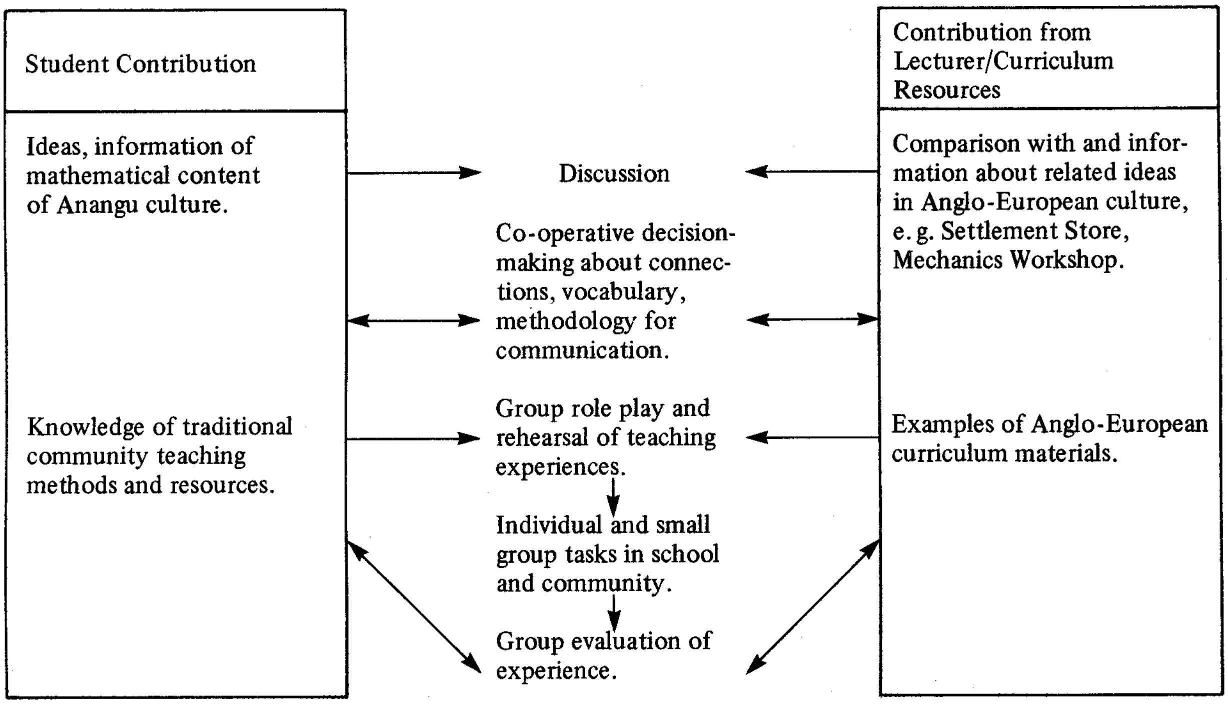
Measurement
Modules 16–20

16. Measurement of length and distance.
17. Measurement of area.
18. Measurement of volume.
19. Money – how much?
20. Money – equivalence, i. e. how much work? how much pay? how much money? how much food?

The experiences provided for students during the course have been chosen so that the links between mathematical ideas as expressed in traditional Anangu culture, the settlement community, Anglo-European culture and the school curriculum are emphasised.

More modules are constructed to promote an exchange of ideas between the distinctly different conceptual frameworks of the two cultures. The diagram below shows some of the ways in which this occurs.

Figure 3



At all points emphasis is placed on practical experimentation and application of ideas in the community and the local school. The teaching staff at Ernabella school has been most supportive of the programme and is enthusiastic about allowing students to expand their practical experience within the school as their skills increase.

Communication Between Cultures

During the development of this course it has been necessary to pay considerable attention to the communication difficulties of a bilingual programme. For some of the reasons expressed in the previous section, language difficulties are *particularly evident* in the mathematics area. Barbara Sayers (1983) suggests that language difficulties are a particular problem in the teaching of mathematics. She goes on to describe her experience at the Wik-Mungkan community at Aurukan:

«I have understood what was said in terms of understanding the linguistic aspects of the language, but I have not understood the message they encoded. Such messages are incomprehensible because I did not understand the presuppositions on which they were built, nor the Aboriginal concepts which were involved. To sum up, I could understand what was said but not what was meant.» (p.3)

The reverse situation occurs all too often when mathematics is taught in Aboriginal schools.
Some proficiency in English has been required for selection to the ANTEP course. However, on-site experience at Ernabella suggests that:

1. Bilingual presentation of materials is essential.
2. Articles and workbooks should include English and Pitjantjatjara translations.
3. Participant responses were always in Pitjantjatjara with the exception of one person.
4. Difficulties will be experienced in translating some English words into Pitjantjatjara particularly in the field of mathematics.

The course has been designed with considerable emphasis on an activity-based process/discovery learning design to maximise the possibility of concensus about the *meaning* of language generated by students (in either language).

It has also seemed appropriate to maximise the superior visual/spatial skills of the Anangu people, and the precision of the Pitjantjatjara language in this respect, by illustrating concepts by role play or diagram as far as possible. For example, in Module 7 of Teaching Mathematics I, posture, which is an effective and important means of communication within the Anangu communities, is used extensively in an action-based activity to convey ideas about seriation. During the development of the course students will be encouraged to develop techniques for using graphic displays to convey relationships.

The Ernabella school has an Apple computer. Work has already been done by Klich[2] in converting spatial games known to community elders into a form suitable for presentation on a video screen. The prospective students have already expressed much inter-

2 Personal correspondence.

est in learning to use this computer. The development of cultural symbolism through the use of Apple Logo software seems an extremely promising medium for the introduction of Euclidean notions of geometry and sequenced procedural skills.

It has become evident that there is insufficient information available about the use of Pitjantjatjara language in relation to mathematical concepts. In any case there has been some suggestion from members of the community that language development in the vernacular is somewhat depressed among settlement children. It seemed advisable to include in the course some action-based research for students directed at the collection of information about the ways in which Pitjantjatjara language is used to describe mathematical ideas.

This seemed an important aspect of their learning because:

- On-location experience with Aboriginal teacher aids suggests that their assistance of children even when expressed in the vernacular consists of, often incomplete, attempts to translate Anglo-European ideas rather than the out-of-school modes of expression. Confusion results.
- The procedure seems likely to provide experiences that will heighten student awareness of mathematical ideas and concepts within their own culture.
- Eagleson et al. (1982) suggest that Aboriginal English is a restricted code. It is restricted, not in the sense that it is inferior but because it has been developed as a means of communicating ideas derived from Aboriginal world views. The syntax of this fomm of English often follows that of the vernacular. Without further development and clarification it is usually not an appropriate means of communicating mathematical ideas from AngloEuropean cultures.
- The collection of information of this kind seems likely to heighten the awareness of both students and educators of points where confusions about mathematical concepts is likely to arise.

The inclusion of these types of exercises in the course was confirmed as a useful innovation by David Wilkins, linguist, of Yipirinya school in Alice Springs. The teachers at that school have found a similar approach most useful, especially for mathematics. Prospective students were enthusiastic about the idea since it affirms their cultural expertise and provides opportunities for consultation with community leaders about precise vocabulary.

The procedure as it has currently been developed involves collecting taped information of children and adult language usage to describe certain situations. For example, a child may be blindfolded and directed through a maze of carefully placed obstacles by the rest of the group to elicit information about vocabulary and syntax connected with location and direction. The school linguist, on-site lecturers and students then use the collected information as a basis for language development in the vernacular and as a source of information about conceptual differences between cultures. For example, Anglo-Europeans tend to describe direction in terms of Left and Right, many Aboriginal groups use the four points of the compass.

There does not appear to be a set policy for bilingual teaching in the Ernabella school. In general, the linguistic resources available and the language competence of children in either language govern the level of verbal discourse. In a bicultural context it is necessary to actively affirm language registers that are appropriate for discourse about science, technology and mathematics. This register of English language is not normally evident in the language-arts curricula of Australian primary schools. The teacher training experience provided in the courses described above, emphasises the use of small group co-operative tasks and elaborated ideas as logical and meaningful communication. It is hoped that these will provide students with the necessary skills to allow for needed curriculum change and a rationale for the appropriate use of first the vernacular and then English as a useful medium for instruction in mathematics.

Conclusion

The emphasis in the teacher training course described above has been in the development of a rationale for connecting the conceptual frameworks (with respect to mathematics) of two very different cultures.

Experiences have been provided to increase student awareness of the mathematical ideas within Anangu culture. Strategies have been suggested and opportunities provided for the development of a rationale for teaching procedures that will assist children as they move from one cultural context to another.

To this end, the course has been constructed on a process model where information is collected and students are encouraged to play an active role in the decision-making about outcomes. Language development in both English and the vernacular will be an important factor in this process.

The visual/spatial knowledge and the heightened awareness of relationships that are characteristic outcomes of Anangu culture will be used in teaching procedures that emphasise these aspects of mathematics rather than demanding prior mastery of incomprehensible algorithmic procedures.

It is to be expected that there will be some problems to be negotiated as the course proceeds. At this stage, however, it seems that success, in terms of student competence as teachers and a more appropriate learning environment for the mathematics curriculum in Anangu schools, may well depend on the extent to which students are enabled to actively participate in building bridges between the two cultures for themselves. Only then will they become truly competent as teachers in a bicultural context.

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Appendix

Module 7

Comparison II — (comparing people)

Lecturer's Notes

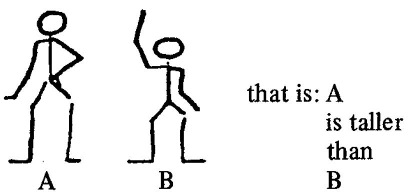
The purpose of this exercise is to expand direct comparison according to one attribute to seriation strategies. The use of people appears appropriate since limits on behaviour and location between different members of the community are likely to be familiar ideas.

You will need: large sheets of paper and felt pens.

Student activity: Explain to students that the following activity is one way of *ordering* by comparison of an attribute using personal orientation to show a *relationship.*

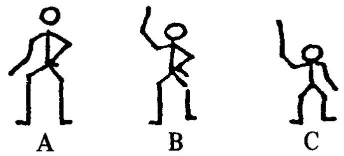
Tutorial Session

1. a) All students to compare heights in the following way. As two people approach, the taller person turns side on and put hand on hip, the shorter person turns side on and puts hand in air
   e.g.

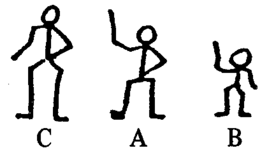


   b) Practice this until all possible pairs have been tried.

   c) After two students have approached each other, as in the above diagram, a third approaches. If he is shorter than A and B he stands next to B side on with hand in air. B places hand on hip facing C.
   e.g.

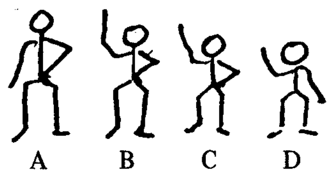


If he is taller than A the following arrangement is made:



   d) Students should try different patterns using the rule that one can only have one person on either side, e. g. a person with both hands on hips can only be approached by two shorter people, and a person with both hands in the air can only be approached by two taller people.

   e) Change the rule so that when both hands are used one should be up in the air and one on the hip.
   e.g.



   The resulting arrangement should be as above. A fifth person approaching the group may:
   - join at the tall end if he/she is the tallest;
   - join at the short end if he/she is the shortest.
   Find a position between two people in the line so that he/she stands appropriately with one hand in the air and one hand on hip.

2. Discuss the activity. Note the similarity of hand on his shape to > symbol used in mathematics. Seek suggestions from students for other actions or postures that may be suitable. How else might people be compared/ordered? Age?, kinship?, totem?, weight?
3. Devise suitable ways of recording results or ordering activities. How can we show the relationship between people?
4. Discuss ways of using this activity and this type of experience when teaching children to order objects according to attributes such as smoothness, length, weight. How can the relationship between objects be shown?

Practicum

The group should devise an ordering lesson suitable for young children (seek modification and improvements). This activity should be carried out with a small group of children (< 10). The results should be reported next session.

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