GPS AND RELATIVITY: AN ENGINEERING OVERVIEW

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Abstract

We give and explain in detail the formulas for the relativistic corrections to be implemented in high-speed aircraft, or when using other satellites in connection with GPS, or when using GPS from another satellite. We explain how to use these formulas in various scenarios, give numerical examples, and itemize the pitfalls to be avoided by (for example) receiver manufacturers.

INTRODUCTION

The Operational Control System (OCS) of the Global Positioning System (GPS) does not include the rigorous transformations between coordinate systems that Einstein's general theory of relativity would seem to require – transformations to and from the individual space vehicles (SVs), the Monitor Stations (MSs), and the users on the surface of the rotating earth, and the geocentric Earth Centered Inertial System (ECI) in which the SV orbits are calculated. There is a very good reason for the omission: the effects of relativity, where they are different from the effects predicted by classical mechanics and electromagnetic theory, are too small to matter – less than one centimeter, for users on or near the earth. However, a new class of users, who employ satellites that obtain time and position in space from GPS, cannot be satisfied with the approximations in the current OCS. Furthermore, because those approximations have not been publicly analyzed and presented, there is much confusion in the GPS literature. Eminent scientists have been divided amongst themselves, wondering whether the OCS software does not need to be rewritten, expecially since the Department of Defense is now requiring that the current specifications – 6 meters in User Range Error (URE) – are to be tightened under the Accuracy Improvement Initiative (AII). In this paper, we compare the predictions of relativity to those of intuitive, classical, Newtonian physics; we show how large or small the differences are, and how and for what applications those difference are large enough to make it necessary to correct the formulas of classical physics.

RELATIVITY: THE FORMULAS

If two observers determine what intuitively we call the same quantity – the distance between two points, or the time interval between two events – they will measure different lengths and times, if (1) they are moving with respect to each other, (2) one is higher or lower than another in a gravitational field, or (3) one is accelerating with respect to the other. Users of GPS

encounter all three effects, and should correct their measurements accordingly, by formulas which we now explain.

The Velocity Effect (Lorentz Contraction)

Case (1) is the province of special relativity. In classical physics, if a transmitter in a vacuum sends out a radio frequency F in its own frame of reference, and if a receiver is moving with velocity v at an angle A to the beam coming from the transmitter to the receiver, then the signal will be received at the Doppler shifted frequency f, where $f < F$ if A is less than 90° (receiver moving away from the transmitter). Now, if the transmitter and receiver are replaced by two units of hardware each of which both transmits and receives, each unit will measure the other's frequency as "too low." We accept intuitively the fact that there is no real paradox here. If the two units were colocated, then if one ran slower than the second, the second must run faster than the first; but if they are not colocated, what an observer measures depends not only on the hardware, but on the state of motion between them; and each observer perceives the motion the same way: "that other is receding from me at a rate $v \cos(A)$." If our problem is to measure f (which is what we observe), and to calculate F (which we must infer), then in classical physics

$$F = f \left(1 + \frac{v}{c} \cos(A) \right) \tag{1}$$

where c is the speed of light, or radio, in free space. If the receiver is moving at right angles to the line of sight, $\cos(A) = 0$, and $F = f$; there is no Doppler shift. But the relativistic equivalent of the same formula is

$$F = \gamma f \left(1 + \frac{v}{c} \cos(A) \right) \tag{2}$$

where γ is defined to be always equal or greater than one:

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

The γ formula looks like the formula for going from sine to secant in ordinary trigonometry, and for good reason. The effect of special relativity (see [1]) is a coordinate rotation through an angle, the arctangent of iv/c. There is a related angle, a real number P, such that $\sin(P) = v/c$, and $\gamma = \sec(P)$; this provides a convenient way of calculating γ. Like the secant, the γ factor (1) is very nearly equal to one for small angles (or v/c), because (2) it increases with the square of the angle v/c for small angles.

Notice that, even if $\cos A = 0$, the Doppler shift is no longer zero, but γ; and so the frequency as received is lower than the frequency transmitted. Since we naively expect, from intuition and from classical physics, a Doppler shift of zero, the usual way to describe the relativistic result is to say that the moving transmitter's frequency standard or clock – moving in the receiver's frame of reference – appears to be running slow. Again, as in classical physics, each observer can consider himself at rest and the others to be moving; and it is no paradox that each measures the others as "too slow." The relativistic formulas defy our intuitions of time and space, but not the basic principles of logic; they can stand.

Now, the angle A should be corrected for aberration. From either a classical or a relativistic point of view, aberration is the change in the apparent direction from which a wavetrain appears to be coming because of the observer's velocity. In Eqs. (1) and (2) above, the cosine of the angle A should be calculated in the observer's frame of reference. If A' is the angle that is calculated in (for example, the so-called "Earth Centered Inertial" or ECI system of coordinates, then the relativistic equation to go from cosine of A' to the cosine of A is

$$\cos(A) = \frac{\cos(A') - \frac{v}{c}}{1 - \frac{v}{c}\cos(A')} \tag{4}$$

and this equation is exact. The reader may wonder why the omnipresent γ factor does not appear. It would, in the formulas for $\sin(A)$ and $\tan(A)$. However, we need $\cos(A)$, and the γ factor cancels out. (For the derivations of all the above equations, see [1].)

So then, to calculate the Doppler correction by which we correct the received frequency from a GPS satellite – for example, in crosslink ranging from one SV to another – we first calculate the relative speed of the transmitter to the receiver in the ECI frame, and the angle A' between relative velocity and the line of sight between transmitter and receiver, also in the ECI frame. We then apply Eq. (4) to obtain $\cos(A)$, and then Eqs. (2)-(3) to transform the observed frequency f to the true transmitted frequency F, in the transmitter's frame of reference.

Gravitational Effect

We now turn to Case (2) above, where the transmitter is higher or lower than the receiver, in a gravitational field. If a photon (whether of light or of radio) falls into a potential well, it is shifted to a higher frequency; climbing up, it loses frequency. The quantum acts like a material particle, gaining kinetic energy as it falls through the gravity field. In quantum mechanics, it is shown that the energy of the photon is $h f$, where h is Planck's constant and f is the frequency. If it were a material particle, then from special relativity its total energy would be $m c^2$, where the kinetic energy is included in the increase of the mass m from the rest mass m_0. Since $h f$ is equivalent to $m c^2$, the mass m of the particle must correspond to $h f/c^2$ of the photon. Then, since the change in potential in going from one point to another is the change in energy of the particle per unit mass, that correspondence requires that

$$\Delta(hf) = m \, \Delta\phi = - \frac{hf}{c^2} \, \Delta\phi \tag{5}$$

or

$$\frac{\Delta f}{f} = - \frac{\Delta\phi}{c^2} \tag{6}$$

In the earth's gravitational field, if a radio signal passes from a satellite at distance R_{sv} from earth's center to a receiver (whether on another satellite or on the ground) at distance R_r, a good approximation for the frequency shift correction, to transform from the frequency measured on the ground to the frequency transmitted by the satellite, is

$$\frac{\Delta f}{f} = \frac{GM}{c^2} \left(\frac{1}{R_{sv}} - \frac{1}{R_r} \right) \tag{7}$$

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So to the Doppler shift given by Eqs. (2), (3), and (4) we add the frequency shift from Eq. (7). If R_{sv} is greater than R_r, the correction is negative, and so by adding it to the received frequency f we obtain the frequency F of the transmitter in its own frame of reference.

Acceleration Effect

This is the correction to be made if an observer (= the receiver) is accelerating. In fact, the receiver, whether in an SV or on the ground, is certainly accelerated; it is either in free fall in orbit or subject to centripetal acceleration on the rotating earth. Acceleration effect can be absorbed in the gravity potential. The formal justification for doing so is the Principle of Equivalence in Einstein's general theory of relativity, which is best explained by an example. Suppose that a photon of light is radiated across a chamber through a distance r from wall A to wall B, along a line perpendicular to both walls. Suppose that the whole chamber is accelerating along this line, in the same direction as the photon is moving, by g meters/second squared. Define the initial velocity as zero at the instant that the photon is radiated; we are entitled to do this by the principles of special relativity. Now, by the time the photon arrives at the opposite wall, that wall is receding at a velocity $v = -gt$, where t is r/c, and as measured by a receiver on the receding wall the photon will be Doppler-downshifted by an amount (in classical approximation)

$$\frac{\Delta f}{f} = -\frac{v}{c} = -\frac{gt}{c} = -\frac{gr}{c^2} \tag{8}$$

But now consider a corresponding scenario in which the chamber is not accelerated, but in which a gravitational field produces a local acceleration due to gravity, g. As we have seen (Eqs. (5)-(7) above), the proportional frequency shift is the difference of gravitational potential divided by c squared. By the Principle of Equivalence, the product gr in Eq. (8) corresponds to this potential difference, and we can allow for the observer's acceleration by modifying the gravity term.

But we see that there is no need to do so, if we remember what this modification would be. The $-gt/c$ of Eq. (8) is simply the correction to the Doppler shift due to the change in the receiver velocity during the signal propagation time – in our example, the time it takes the photon to traverse (r). But if we compute the Doppler effect by Eq. (2), using the relative velocity between transmitter and receiver at the time of reception of the signal, we include the effect of Eq. (8) automatically. Only if we were to label signal events at the transmission time would we need to include Eq. (8) explicitly. Among GPS users, hardly anyone does so. If, for any reason, it is necessary so to do, rewrite Eq. (8) as

$$\frac{\Delta f}{f} = -\frac{\vec{a} \cdot \vec{r}}{c^2} \tag{9}$$

where $\vec{a}$ is the user's vector acceleration in an inertial frame (e.g, the ECI), and $\vec{r}$ is the vector from transmitter to receiver – at either transmission or reception time, in the "weak field approximation." This is in the frequency domain; integrate, for timing receivers.

Summary of Corrections To Be Made

Here is the summary. In general, to correct the frequency measured by a receiver for all relativistic effects, to obtain the frequency at the transmitter, we perform the following steps.

(i) Obtain the velocity $\vec{V}$ of the transmitting satellite, and the velocity $\vec{v}$ of the receiver, in an ECI coordinate frame, for the times of reception. Calculate the angle between the receiver's velocity vector and the line from transmitter to the receiver, in the ECI frame. That is the angle A' of Eq. (4).

(ii) Apply Eq. (4) to obtain the corresponding angle A in the receiver's frame of reference.

(iii) Calculate the magnitude of the velocity of the GPS satellite with respect to the receiver, in the ECI frame. That is simply the magnitude of the vector difference $\vec{V} - \vec{v}$. Using Eq. (3), calculate γ.

(iv) Calculate the $\Delta f/f$ frequency shifts using Eqs. (2) and (7), and add them together. This is the total correction to be added to the received frequency to obtain the transmitted frequency.

To obtain the correction to add to the transmitted frequency in order to obtain the received frequency, perform the above steps and reverse the sign. To convert to the time domain, integrate, and assume any convenient zero of time. Without a timing receiver observing four satellites, one cannot synchronize to GPS time.

Since GPS receivers work in the time and not in the frequency domain, they handle the velocity, gravity, and acceleration shifts differently than described above. First, each GPS space vehicle (SV) clock is offset from its nominal rate by about -4.45×10^{-10} (= -38 microseconds per day) to allow for the relativistic offsets between the differences between the SV and the ground. Of this -38 microseconds per day, about -45 are due to the gravitational potential difference between the SV at its mean distance and the earth's surface, and $+7$ to the mean SV speed, which is about 3.87 km/sec. To this mean correction, each receiver must add a term due to the eccentricity of the GPS orbit. It can be shown that this effect produces a variation in the SV clock, as seen from the earth, of

$$\Delta T = -2 \frac{\vec{R} \cdot \vec{V}}{c^2} \tag{10}$$

where $\vec{R}$ is the vector of position of the SV from the ECI, and $\vec{V}$ the velocity vector. This is the equation given in ICD-GPS-200. It is appropriate for users on or near the earth's surface, but not users in space, who should apply the frequency correction equations given above, or their integrals to transform to the time domain.

STICKY WICKETS

We now turn to the misunderstandings that have arisen over the 15 years or so that GPS has been in operation. Most of the disagreements have semantics for a father. A few arose from the mathematics of relativity, in going from a rigorous treatment to the approximations that are found sufficient for the practical use of GPS.

The Newtonian World of the Operational Control System

The GPS Operational Control System (GPS) corrects the pseudoranges measured by its Monitor Stations for the sum of the gravitational and the velocity effects as discussed in the previous section, but computes the velocity effect only for the mean orbital speed of the satellite, which is about 3.87 km/sec. The question has been raised: why doesn't the operational software calculate the velocity effect using the speed of each satellite relative to each Monitor Station, which varies

sinusoidally as the Monitor Station is carried around the rotating earth? In principle, it should, but the effect is negligible for two reasons: (1) the nature of the pseudorange measurement; and (2) the size of the effect.

The pseudorange measurement can be regarded from two points of view. On the one hand, pseudorange is simply range, once the clock offsets are removed. As described in the previous section, the ranges measured by a moving observer are foreshortened by the γ factor. For an SV speed of 3.87 km/sec, $\gamma - 1$ is 8.33×10^{-11}. The range is typically about 30,000 km. The error incurred neglecting the γ factor is 30,000 km multiplied by 8.33×10^{-11} – that is, 2.5 millimeters. Close enough for government work.

On the other hand, the pseudorange measurement should be equivalent to accumulated Doppler, once the ionospheric delays are removed. If the MS accumulated Doppler measurements over a pass, then, since the OCS assumes Newtonian physics, it would use Eq. (1). In the more nearly correct world of special relativity, we should use Eq. (2), which differs from Eq. (1) by the γ factor. Not only the velocity term, but the observed frequency f, is multiplied by γ. And γ is not constant, because the rotation of the earth (up to 0.465 km/sec at the equator) vectorially adds to the 3.87 km/sec of the satellite when we compute the satellite speed relative to a Monitor Station. Corresponding to a possible v of 3.87 ± 0.465 km/sec, γ can vary from $8.33 \times 10^{-11} \pm -1.88 \times 10^{-11}$. The constant part is absorbed in the mean relativistic offset described above, but the variable part would alias into the accumulated Doppler range. For example, over one hour = 3,600 seconds, the error could be 1.88×10^{-11} times 3,600 seconds times c = 20.3 meters. Clearly, this would be a major source of error in the Operational Control System.

But this error would be incurred only if the station clocks were independent of the GPS satellite clocks, each MS keeping its own time. That's not the way the OCS works. Station time is estimated in the Kalman filter together with the SV clocks, and each MS clock is effectively being updated continuously by the satellites. The station clock is used only to bridge the gap in time between measurements; but since several satellites are always in view, these measurements are virtually instantaneous, except for the different signal propagation times from different satellites. These times are about 0.1 sec, and, multiplying by γ, once more we derive an error of about 2.5 millimeters.

However, this happy result depends entirely on the Kalman filter. At present, the filter is very "springy" – that is, it has a short time constant, because it is adjusted to absorb ephemeris errors which can change rapidly (for example, at the onset of eclipse). Station clocks, satellite clocks, and ephemeris parameters are all allowed to move together, and so Monitor Station time is controlled entirely by the satellite clocks. (If several satellites are in view, the Monitor Station clock is mathematically redundant, and in practice it drops out of the solution.) But if the filter were retuned to be very "stiff," with long time constant and reasonable weights assigned to the Monitor Station clocks, then the appropriate time over which the relativistic effects would act would no longer nearly equal the signal propagation time, but would be several hours or days; and then the error due to the neglected γ factor would approximate that incurred in the hypothetical accumulated doppler scenario – many meters.

In principle, the critics of GPS in the relativity debate have not been completely wrong. The neglected γ factor could hurt us. The OCS software should be reformulated. Nevertheless, in practice, neglect of relativity does not now contribute measurably to the GPS error budget, as the OCS software is currently configured.

The ECI Coordinate System

It has been said (e.g., [3] and [4]) that the Operational Control System (OCS) of GPS makes all calculations with respect to an ECI ("Earth Centered Inertial") frame of reference. This is a perfectly defensible statement, depending on how one chooses to describe the GPS time scale.

In actual practice, GPS operates in a mixed coordinate system: the spatial coordinates are ECI, but the time rate is appropriate to observers on the surface of the rotating earth, that is, in the ECEF. GPS time is steered as closely as possible to U.S. Naval Observatory time (UTC[USNO]), which in turn is steered closely to International Atomic Time (TAI = Temps International Atomique). By an important theorem of general relativity, ideal frequency standards on the rotating geoid run at the same rate, because the effect of the earth's rotation (by which clocks on the equator would run more slowly than clocks at the poles) is cancelled by the earth's gravitational potential (since clocks on the equator are farther from the earth's center, and so at higher potential, than clocks at the poles). So a frequency standard anywhere on the earth's surface runs at the same rate as one at either pole, and is offset by a constant rate from a (hypothetical) clock at the earth's center. By the definitions adopted by the International Astronomical Union (IAU), the ideal time that GPS, UTC, and TAI time scales realize as nearly as possible is called Terrestrial Time (TT), and the time interval unit of TT is the Standard International (SI) second on the geoid, also called the coordinate second. Then, in at least one sense, the time scale of GPS is defined in the ECEF coordinate frame. Also by Recommendation T1 of the Sub-Group on Time of the IAU Working Group on Reference Systems, "coordinate times in (a) non-rotating reference system having ... spatial origin at the geocenter ...(is) designated as Geocentric Coordinate Time (TCG)"; this definition was adopted by the IAU in its General Assembly in 1991. Therefore, if GPS were to operate in the ECI coordinate system in the strict sense, its time scale would be TCG, which is offset from TT, etc. by the potential difference between earth's center and its poles. This GPS does not do.

If a satellite user of GPS were naively to map all measurements "to an ECI frame," using Geocentric Coordinate Time (TCG), an error would be incurred of roughly 60 microseconds per day.

However, there is a sense in which the TAI time coordinate is related to the ECI. Although all ideal frequency standards run at the same rate on the geoid, clocks cannot be synchronized unless some sort of signals pass between them. Because of the earth's rotation, when clocks on different continents are compared by travelling frequency standards, satellite signals, or other means, corrections must be made for the relativistic Sagnac effect. To calculate these corrections, one must know the rate of the earth's rotation, that is, the angular rate of the ECEF frame relative to the ECI. Implicitly, then, in the process of forming TAI, to which GPS time is steered, reference is made to the ECI.

Missing Relativity Terms?

Oversimplifications such as in [4], which disseminated the mistaken notion that GPS time is calculated "in the ECI," ignoring the earth's rotation, misled Steven Deines, in his paper entitled, "Uncompensated relativity effects for a ground-based GPS receiver."[5] Deines argued that

> The current ...GPS relativity corrections were based on
> an Earth centered inertial reference frame. The derivation
> assumed [that] the receiver obtains inertial GPS coordinate

time from the satellites. However, the receiver has been
treated tacitly as being stationary in the inertial frame...
relativity effects for a ground-based receiver include
gravity and Earth rotation. Airborne GPS receivers have
larger effects, and spaceborne GPS receivers have the worst
uncompensated relativity effects.

Deines used equations from Robert Nelson's work (see [6]) in an attempt to show that corrections depending on the earth's rotation should be added to those incorporated in existing GPS receivers according to ICD-GPS-200. A complete critique of Deines' derivations has been made by Neil Ashby.[7] Deines overlooked cancellations among several of his terms, by which the Nelson expressions reduce to very nearly the formulation of ICD-GPS-200.

Nelson shows ([6], p. 21) that the transformation between a time increment dt' expressed in a rotating and/or linearly accelerated reference frame and a time increment dt in a nonaccelerating frame is given by

$$dt \ = \ \gamma \ [\ 1 + \frac{\vec{W} \cdot \vec{\rho}}{c^2} + \frac{\vec{v}}{c^2} \cdot (\frac{d\vec{\rho}}{dt'} + \vec{\omega} \times \vec{\rho}) \ ] \ dt' \tag{11}$$

where $\vec{W}$ is the acceleration, $\vec{\rho}$ the position vector of a GPS receiver, $\vec{v}$ its velocity, and $\vec{\omega}$ the earth's angular rotation rate. If the ground station is motionless with respect to the earth's surface, then the acceleration vector $\vec{W}$ is determined by $\vec{\omega}$ and by $\vec{R}$:

$$\vec{W} \ = \ \vec{\omega} \times (\vec{\omega} \times \vec{R}) \tag{12}$$

and also the velocity $\vec{v}$:

$$\vec{v} \ = \ \vec{\omega} \times \vec{R} \tag{13}$$

Then one may rewrite Eq. (11), letting the origin from which $\vec{\rho}$ is measured to be the GPS receiver, of which the position in ECI coordinates is the vector $\vec{R}$. Eq. (11) now becomes

$$dt \ = \ \gamma \ \{ \ 1 \ + \ \frac{[\vec{\omega} \times (\vec{\omega} \times \vec{R})] \cdot \vec{\rho}}{c^2} \ + \ \frac{(\vec{\omega} \times \vec{R}) \cdot (\frac{d\vec{\rho}}{dt'} + \vec{\omega} \times \vec{\rho})}{c^2} \ \} \ dt' \tag{14}$$

As Nelson says, terms cancel in Eq. (14) to give a very simple result. The cancellation is as follows. As is shown in vector algebra,

$$\vec{A} \cdot (\vec{B} \times \vec{C}) \ \equiv \ (\vec{A} \times \vec{B}) \cdot \vec{C} \tag{15}$$

Therefore, we can write

$$(\vec{\omega} \times \vec{R}) \cdot (\vec{\omega} \times \vec{\rho}) \ = \ [\ (\vec{\omega} \times \vec{R}) \times \vec{\omega} \ ] \cdot \vec{\rho} \ = \ - \ \vec{\rho} \cdot [\ \vec{\omega} \times (\vec{\omega} \times \vec{R}) \ ] \tag{16}$$

and Eq. (14) becomes

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$$dt = \gamma \, [dt' + \frac{1}{c^2}(\vec{\omega} \times \vec{R}) \cdot d\vec{\rho} \,] \qquad (17)$$

Eq. (17) "is just what one would expect by a Lorentz transformation from the center of rotation to the instantaneous rest frame of the accelerated origin" ([6], p. 23). Except for the leading γ factor, it is the same as the formula derived in classical physics for the signal travel time from the GPS satellite to the ground station. As we have shown, introducing the γ factor makes a change of only 2 or 3 millimeters to the classical result. In short, there are no "missing relativity terms." They cancel out.

8 REFERENCES

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Questions and Answers

STEVEN HUTSELL (USNO): You and I have been talking about this intermittently over the phone over the past couple months or so. Obviously, the question of trying to make the best utilization of the ephemeris and solar process noise values in the MCS Kalman filter is something to consider to try to best compensate for this. I have to plead ignorance on my behalf because I haven't done any studies really or any simulations. I'm wondering if you or anyone else at Aerospace has pursued this with different choices for simulation.

HENRY FLIEGEL: No, I told our sponsor that for the limited amount of money that he paid us, he gets about this much and no more. It would really take quite a bit of effort to simulate in detail what that Kalman filter is doing. And frankly, I don't think I have the expertise myself, but of course there are other people at Aerospace Corporation that do. But we would have to be turned on by the Air Force or by somebody that has a charge number.

GERNOT WINKLER (INNOVATIVE SOLUTIONS INT'L): I have two comments or questions to ask you for your reaction to that. Number one, you said that the Kalman filter has no place to put these residuals; or better, that they will not be able to show up in the ephemeris part because it does not agree with the observations.

But there's one parameter which is very sensitive, and that's the frequency offset in the clocks. You have any error in the frequency offset in the clocks, it will, of course, accumulate since the clock offset will be larger proportional to the time offset since the last upload. But that's the most sensitive part, and I'm not sure if your conclusion is correct in that regard.

Number two, for most time users who use GPS, they're exactly those which you have excluded from your consideration, because they do observe the satellites in 30-minute passes; and an apparent frequency offset of 10 to the minus 10th in 30 minutes produces an offset of more than 10 nanoseconds. And that's really the explanation for the bowing effect, isn't it?

HENRY FLIEGEL: I wonder if it is. Let me deal with your last point first. Of course, we are concerned that some users may in fact be affected by this. That's why we're putting out the report. And I don't know whether perhaps the bowing effect is at least part due to this, other parts, ionosphere and what have you.

To get back to your first point, remember, though, that we're continually reporting the measured offset of each clock in the system from GPS time. So we're absorbing and reporting the very thing that you're talking about.

GERNOT WINKLER: Excuse me, yes, of course, during the time when your Monitor Station sees the clock. But you're uploading predicted information. It is here that the residual error in the frequency offset will become important.

HENRY FLIEGEL: It may. But remember, the effect repeats – at least, in my calculations – day after day after day. So essentially, if you take a mean value for a given day, probably the predictions will not be too badly impacted.

My other point was I said only it would be very difficult to alias clock error into ephemeris to any large extent. But I'm not saying that the clock error has just disappeared.

CARROLL ALLEY (UNIVERSITY OF MARYLAND): I'm happy to hear Henry agree at the end that one should look very carefully at the details of the relativistic physics. And it has been my contention that if one does that, software changes alone in the current system might bring these UREs down below the 1-meter level, and get actual user performance down at the one to 2-meter level with the existing system. And it has been very difficult to get into the

system and to get the adequate support to do these studies. But I'm very happy that there seems to be an agreement that one should do this. It is not an expensive matter compared to the 10 or 12 billion that's being invested in the GPS.

I would like to make another point. When one looks at differential GPS, the correction that needs to be made primarily is the difference between the radial distance in the ephemeris and the time reading on the satellite. And I believe this comes in because of a mix-up, or aliasing if you will, between these two quantities in the iteration procedure that the Kalman filter is following. And that if one perhaps does the explicit recognition of the special relativistic effects – I mean, it took a long time to get general relativity down properly, but I think that is more or less correct now. But it's the absence of any explicit acknowledgment of special relativistic effects due to the speed of light being the same whenever measured by an observer, leading to the relativity of simultaneity and the associated Lorentz transformation physics – there's nothing of that at all modeled in the current system, and I think it should be. Thank you.