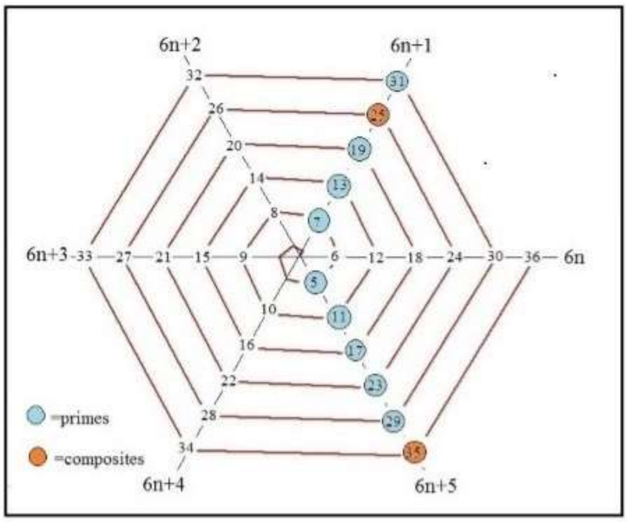
HEXAGONAL-INTEGER-SPIRAL AND PRIMES

About a decade ago I came up with two new concepts in Number Theory. The first of these was to note that the standard Ulam approach for plotting integers makes primes appear in a random manner. This struck me as strange and I began searching for a transformation which would get around this difficulty. After some effort I found a way which led to the following hexagonal integer spiral representation-

HEXAGONAL INTEGER SPIRAL



Note here that all primes five or greater lie exclusively along just two radial lines N=6n$\pm$1 which emanate from the origin of a hexagonal integer spiral. Since there are also some integers (orange) along these two radial lines we are able to make the following statement –

A necessary but not sufficient condition that a number is a prime is that N=6n$\pm$ 1 and n$\geq$1

Note that 6n+5-6 are equivalent to 6n-1 for such a mod(6) arrangement of numbers.

The second thing we found is that there exits a new non-integer defined as-

f(N)=[sigma(N)-N-1] /N

termed by us the Number Fraction. Here σ(N) is the sigma function of number theory . It represents the sum of all divisors of N including 1 and N. An interesting property of f(N) is that it will equal zero whenever N is a prime. This also means that any number five or greater is prime provided-

f(N)=0 or σ(N)=(N+1)

Thus the number N=2786179021 is a prime since it has f(N)=0 and also σ(N)-1=N.

We want in this article to further explore the properties of primes using the above information. We begin by looking at a long table out to 22 turns of the hexagonal integer spiral. Here are the results-

n	1	2	3	4	5	6	7	8	9	10	11	12
6n+1	7	13	19	---	31	37	43	---	---	61	67	73
6n-1	5	11	17	23	29	---	41	47	53	59	---	71

n	13	14	15	16	17	18	19	20	21	22
6n+1	79	---	---	97	103	109	---	---	127	---
6n-1	---	83	89	---	101	107	113	---	---	131

The dashed elements in this table are composites obtainable via the operation ifactor(N). Note that many of these will follow if the element in the previous turn ends in 9 so that the next turn will end in 5. Also if the term is a square we have a composite.
The computer program(MAPLE) we used which will let us find primes out to N =6*n+1 is-

f:=(sigma(N)-N-1)/N   for n from 1 to N do {n,f,N}od;

For the second case where N=6*n-1 we have-

f:=(sigma(N)-N-1)/N   for n from 1 to N do {n,f,N)od;

A zero present in the triplet {n,f,N} result means we have a prime. Thus the solution triplet {94,0,563} and {81,0,487} means that 487 and 563 are primes. Note that 487 mod(6)=1 and 563 mod(6)=5 meaning 487 is a 6n+1 prime while 563 is a 6n-1 prime. One does not need to start the calculations with n=1. So, to find the nearest prime near turn n=1000 one simply needs to check in the range 1000-5 to 1000+5. Doing so produces the closest prime described by the triplet {n,f,N}={1001,0,6007}. This means p=6(1001)+1=6007. From it we can further state that N lies on the 1001 turn of the spiral. The ratio of N to n will always be close to 6, meaning that there will be six integers along the spiral between the n and n+1 turn of the spiral.
The nearest prime near one million occurs for n=999999 which makes N mod(6)=5 meaning that N lies on the 6n-1 radial line. The triplet is {999999,0,5999993} so the prime equals 5999993 .

You will notice that some of the composites lying along the 6n$\pm$1 radial lines (and marked in orange)are semi-primes S=pq, where p and q are primes. Thus

S=25=5x5 and S=5x7=35 are seni-primes. Such semi-primes must always lie along the two radial lines 6n$\pm$1. So 25 mod(6)=1 meaning 25 lies along the 6n+1 radial line. The semi-prime 35 has 35 mod(6)=5 so it lies along the 6n-1 radial line. Excluding 2 or 3, it allows us to state that-

Any semi-prime S=pq must lie along one of the two radial lines 6n$\pm$1

The value of f for such semi-primes are given by-

$$f=(p+q)/pq$$

and thus lies just slightly above zero for larger Ss. Consider the semi-prime S=9047=83X109. Here we have f=(83+109)/9047=0.0211119… .
To factor large semi-primes S we use the identities-

$$Sf=p+q \quad \text{and} \quad f=[\sigma(S)-S-1]/S$$

Eliminating f then produces the solution-

$$[p,q]=(1/2)\{[\sigma(S)-S-1]\pm\sqrt{[\sigma(S)-S-1]^2-4S}\}$$

Applying this result to S=9047, where our computer yields σ(9047)=9240, we have-

$$[p,q]=96\pm\sqrt{96^2-9047}=96\pm13=[83,109]$$

This very simple factoring continuous to hold out to semi-primes where the sigma function can no longer be produced on my PC in a split second . Using our MAPLE math program we can get values of σ(S) out to about S equal to 20 digit length. An example of this factoring approach is-

1774319431086405772344947305713375666887=
27961320846321079937 x 63456209412934657351

To factor 100 digit long semi-primes S will require finding sigmas for this size. So far no one has succeeded in this endeavor although I think it will be possible to accomplish this in the near future making public keys used in cyber-security obsolete.

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